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Observation of negative group delays within a coaxial photonic crystal using an impulse response method

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Abstract

An impulse response experiment is described which independently verifies the recently observed result of negative group delayed propagation within a coaxial photonic crystal. This result was unexpected when it was reported and has been the subject of enduring controversy because theoretical models predict that negative group delays should not be possible in passive linear media. The impulse response method allows for the determination of both the transmission amplitude and the group delay for narrow-band wave packets over a wide frequency range in one simple experiment. The impulse response results presented here confirm our earlier finding of negative group delays within the band gap region and are in excellent agreement with the more traditional approach of measuring transit times for wave packets. However, negative group delays are observed only over very narrow frequency ranges within the wider band gap suggesting that another interference mechanism is involved.

1. Introduction

Recent experiments [1–3] have demonstrated that the group velocity of a classical wave packet can greatly exceed c, the speed of light in vacuum. Such behavior can be observed in regions of anomalous dispersion associated with strong attenuation in the forbidden transmission regions of electrical, acoustic, or photonic band gap materials. In some cases it has been found that the time it takes for the peak of a wave packet to traverse such a structure can even be negative. This phenomenon has been observed in transparent media with optical gain [4], electronic circuits [5–7], and coaxial photonic crystals [8]. A passive coaxial serial loop structure has also been predicted to exhibit similar behavior [9]. A negative transit time, or negative *group delay*, corresponds to the exiting of the peak of a wave packet from the sample before the peak of the input wave packet has entered the sample. Although this effect is somewhat counterintuitive, it has been shown in the case of electronic circuits that such negative group delayed propagation is not in conflict with relativistic causality [5,10].

Our recent observation of negative group delays in a coaxial photonic crystal [8] is, however, somewhat surprising from a theoretical viewpoint. It has been shown [11] that for a loss-less one-dimensional photonic crystal the group velocity cannot exceed $c/|t(\omega)|^2$, where $t(\omega)$ is the amplitude transmission function for the photonic crystal. Thus, for a loss-less, one-dimensional photonic crystal, negative group velocities should not be observed. Initial attempts by our group to numerically model losses, increased Fabry–Perot interference between the ends of the coaxial photonic crystal, and isolated impedance defects, have resulted in similar findings without any prediction of negative group delays. Similarly, Poirier et al. [12] have shown numerically that negative group velocities should not be possible in any linear passive medium and have questioned our previously reported results. In particular they suggest that the negative group velocities of [8] were due to experimental error and interference and/or distortion caused by the signal generation scheme. In order

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to gain a clearer insight into our earlier experimental results and to address questions of equipment reliability and experimental technique, we used an impulse response method to more fully characterize the coaxial photonic crystal by determining the group delay both inside and outside the forbidden transmission region. The impulse technique eliminates the experimental error associated with measuring the peak position of pulses and does not rely on the pulse generation method criticized in [12]. This experiment illustrates that negative group delays are indeed observed but only over narrow frequency intervals within the band gap. Further, these regions correspond to narrow transmission minima within the forbidden band gap. A number of more recent papers demonstrate that negative group delays are possible in passive waveguide systems. The authors of Ref. [9] demonstrated negative group delays in coaxial cable loop filters. It is difficult to see, however, how this loop mechanism would manifest itself in our experiment. More pertinent are the simulations presented in Ref. [13] in which the author demonstrates negative group delays in a one-dimensionally corrugated waveguide—a system that closely parallels our experimental arrangement. The same author has reported similar results for a two-dimensional system [14].

To better understand negative group delays, we can consider the group velocity expression for an optical wave packet in one-dimension, which is given by

$$v_g \equiv \frac{d\omega}{dk} \cong \frac{c}{n + \omega\left(\frac{dn}{d\omega}\right)}, \tag{1}$$

where n is the index of refraction of the medium. From this expression it can be seen that in regions of anomalous dispersion ($dn/d\omega < 0$), v_g can exceed c and will become negative if

$$\omega\left(\frac{dn}{d\omega}\right) < -n. \tag{2}$$

In this way negative group delays can be achieved.

In the experiments described here, we present a simple method for characterizing a coaxial photonic crystal, a quarter-wavelength stack of coaxial cables which acts like a band stop filter, using an impulse response method that allows us to determine the transmission amplitude and delay time in a single experiment.

2. Impulse response method

The impulse response method is ideal for characterizing the so-called coaxial photonic crystal due to the simplicity of the setup and the accuracy of the results. By recording the time domain information for two pulses, one that has traversed the structure and one that has not, the transfer function of the structure can be obtained which allows for broadband characterization of amplitude and phase information.

Fig. 1 shows the experimental setup used for characterizing the coaxial photonic crystal. A square wave signal was high-pass filtered using a simple RC circuit ($R = 820\ \Omega$ and $C = 68$ pF) to produce the impulse shown in Fig. 1a at the rising edge of the square wave. The values of R and C were selected so that the impulse would contain a broad range of frequencies that extend beyond our range of study (2–16 MHz).

The impulse was sent through the coaxial photonic crystal, which is described later in detail, and the TTL output of the signal generator was used as a trigger for the acquisition of data. Fig. 1b shows the recorded impulse at the oscilloscope. The oscillations after the initial impulse are due to multiple reflections within the periodic structure. A 50 Ω termination resistor was installed

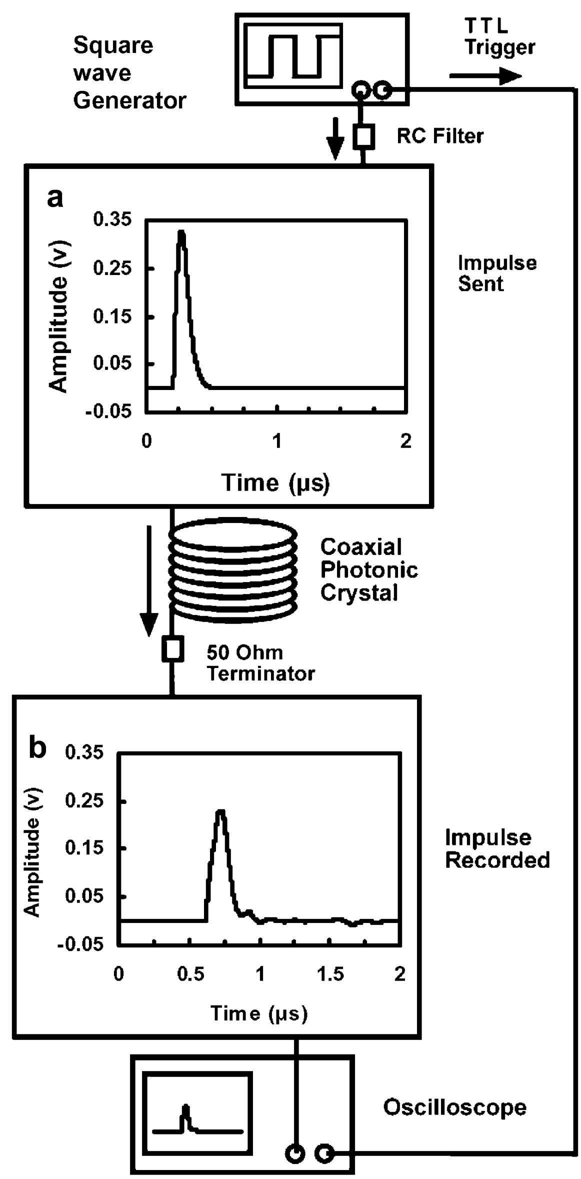

Fig. 1. Experimental configuration for the impulse response method. (a) Impulse entering the structure. (b) Impulse received after traversing the coaxial photonic crystal.

at the oscilloscope's input to reduce back reflections resulting from an impedance mismatch between the last cable of the structure and the input of the oscilloscope.

To determine the transmission function, two time domain signals are required: the impulse response of the sample under test and a reference impulse. In order to obtain the reference impulse we determined the impulse response for a zero-length section of coaxial photonic crystal. To achieve this in practice we replaced the 91.15 m coaxial photonic crystal structure with a 0.20 m piece of RG58 cable. To offset the timing delay of the impulse introduced by having to travel through the 0.20 m cable, the TTL signal's path, as shown in Fig. 1, is also extended by this amount. In this way the oscilloscope's trigger is delayed such that the original impulse appears to have traveled a distance of 0 m. Any losses due to the 0.2 m cable are negligible compared to the losses in the 91.15 m coaxial photonic crystal used here and were not considered.

For the experimental results presented here, the recorded impulse consisted of 22 000 data points and had a time window of 12 μs. An add-and-average option on the oscilloscope (Agilent 54622D) was used to acquire time-domain data with a high signal-to-noise ratio. The number of impulses averaged in a single measurement (typically more than 500) was selected such that sufficient amplitude was acquired even at the lowest transmission point of the band gap. The effect of insufficient averaging could clearly be determined because it led to discontinuities in the phase profile when the data were Fourier analyzed.

The coaxial photonic crystal used for this experiment consisted of 13 sections of alternating RG58 (50 Ω) and RG62 (93 Ω) cable. The cables were cut to lengths of 6.19 m (for RG58) and 7.97 m (for RG62) in order to produce a quarter-wavelength interference filter with a frequency band stop at 8 MHz. The cables were cut to different lengths due to the differing propagation speeds of .66*c* and .85*c* in the 50 Ω and 93 Ω cables, respectively.

3. Impulse response analysis

In the previous section we described the measurement techniques used to determine the voltage amplitudes as a function of time for the reference impulse and for the sample impulse that had traveled through 13 sections of coaxial photonic crystal. By taking the Fourier transforms of this time domain data, we obtain the complex frequency domain data for the two impulses, which are related by

$$\widetilde{V}_{\mathrm{out}}(\omega) = T(\omega)\widetilde{V}_{\mathrm{in}}(\omega), \tag{3}$$

where $T(\omega)$ is the transfer function for the structure and $\widetilde{V}_{\mathrm{in}}(\omega)$ and $\widetilde{V}_{\mathrm{out}}(\omega)$ are the complex Fourier transforms of the reference and sample voltage impulses, respectively. The transmission amplitude is then given by

$$|T(\omega)| = \left|\frac{\widetilde{V}_{\mathrm{out}}(\omega)}{\widetilde{V}_{\mathrm{in}}(\omega)}\right| \tag{4}$$

and the phase is given by

$$\Phi(\omega) = \arg[T(\omega)]. \tag{5}$$

The group delay, t_g, about the carrier frequency ω_0 for a narrow frequency band wave packet that has traveled a distance L through a structure can be defined from the group velocity as

$$t_g \equiv \frac{L}{v_g} = \frac{L}{\left(\frac{d\omega}{dk}\right)\big|_{\omega_0}} = -\frac{d\Phi}{d\omega}\bigg|_{\omega_0}, \tag{6}$$

where $\Phi(\omega) = -k(\omega)L$ is the frequency dependent phase shift. The transmission, phase, and group delay are shown in Fig. 2 for the coaxial photonic crystal described in the previous section. The overlay in Fig. 2b shows a close-up of the phase near the band gap. There are two regions where the slope of the phase becomes positive, one at 7.3 MHz and a second at 8.5 MHz. At these points, the group delay will become negative as is apparent from Eq. (6) and Fig. 2c. The peak of a narrow-band wave packet that contains either of these frequencies should tunnel and emerge from the structure before the peak of the original wave packet has entered the structure's input. The results of phase and group delay measured here show a strong similarity to the results presented in Figs. 3 and 4 of reference [13] in that there are additional dips in these quantities at frequencies within the forbidden band gap.

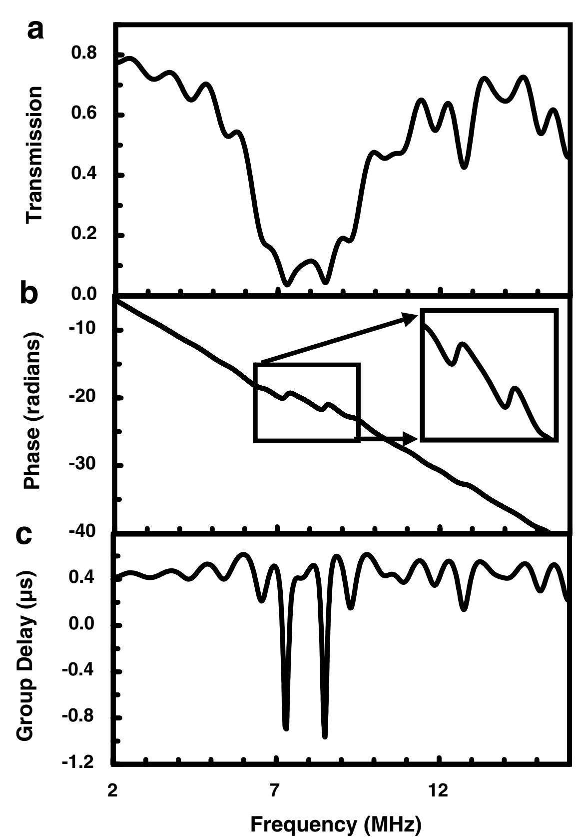

Fig. 2. Results obtained using the impulse response method for a 91.15 m coaxial photonic crystal. (a) Transmission amplitude. (b) Phase. (c) Group delay.

It is at these points in our data that the group delay is enhanced to the point of becoming negative. This correspondence is strongly suggestive that a mechanism similar to that provided in Ref. [13] is at work here.

4. Experimental tunneling results

Finally we compare the above results with those obtained by creating and sending narrow-band wave packets through the structure and measuring the transit times as was done in [8,16]. Two wave packets were considered, one with a carrier frequency of 5.12 MHz and one with a carrier frequency of 7.27 MHz. Fig. 3 shows the envelope function for both wave packets. The high frequency oscillation of the carrier sinusoid is suppressed in the figure for clarity. The top wave packet in the figure corresponds to a carrier frequency of 5.12 MHz with a measured group delay of 0.46 ± 0.02 μs. The bottom wave packet corresponds to a carrier frequency of 7.27 MHz with a group delay of $-0.40 \pm .02$ μs (i.e. the peak of the wave packet exited $0.40 \pm .02$ μs before entering the sample). This corresponds to a speed of approximately $-0.76c$.

Fig. 4a shows the amplitude transmission and Fig. 4b the group delay for the structure as measured using the impulse response method. Overlaid on the transmission spectrum of Fig. 4a is the frequency spectrum of the tunneled wave packet with a 7.27 MHz carrier frequency. Most of the frequency components of the wave packet lie within a region of negative group delay; however, due to the width of the wave packet's frequency domain data, the different frequency components experience slightly different group delays. This is thought to lead to the minor distortion and asymmetry of the tunneled wave packet that can be seen in Fig. 3.

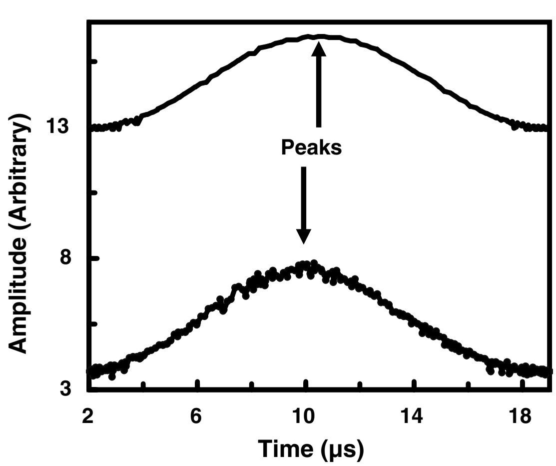

Fig. 3. Envelope functions for two wave packets that have traversed the coaxial photonic crystal. The top wave packet corresponds to a carrier frequency of 5.12 MHz with a measured group delay of 0.46 ± 0.02 μs. The bottom wave packet corresponds to a carrier frequency of 7.27 MHz with a group delay of -0.40 ± 0.02 μs. The carrier frequencies have been suppressed and the upper wave packet has been vertically offset for clarity.

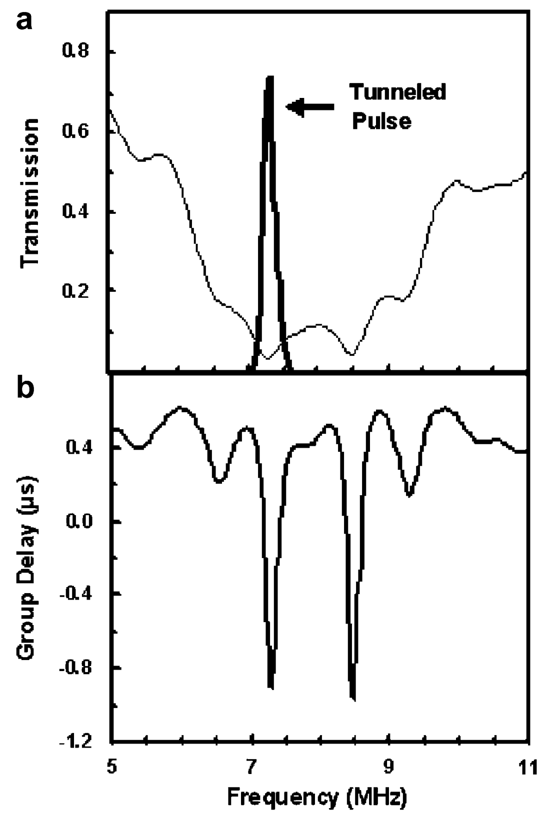

Fig. 4. (a) Frequency spectrum of a wave packet (thick line) with a carrier frequency of 7.27 MHz overset on the transmission amplitude of the coaxial photonic crystal (thin line). (b) Group delay determined using the impulse response method shows an expected negative group delay for the wave packet in part (a).

5. Analysis and conclusions

We have demonstrated an impulse response method capable of determining the transmission function for a coaxial photonic crystal that verifies the existence of regions where wave packets can propagate with negative group velocities. Although this measurement confirms our earlier finding of negative group delays, we still do not know the origin of the sharp features in the transmission function that lead to the narrow bands of negative group delay. We have made attempts to reproduce these results by numerical modeling including the addition of loss terms, increased Fabry–Perot interference, and sparse impedance defects; however, we have thus far been unable to achieve a model that predicts negative group delays.

Although impulse response methods have been used previously to determine transmission and phase information for a variety of physical systems including optoelectronically pulsed antennas in the GHz regime [15], we have taken this method a step further to determine the expected group delays and hence group velocities for a coaxial photonic crystal structure in the MHz regime. We then verified these results using a more traditional method of measuring transit times for wave packets and found excellent agreement between the two methods. The simplicity with which these broadband characteristics can be determined using the impulse response method makes it ideal for such studies.

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