Superluminal time advance of a complex audio signal

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There has been much recent interest in the study of superluminal pulse propagation; however, all previous findings have been extremely limited, only observing superluminal group velocities for simple Gaussian-like pulses. We now report the observation of a superluminal time advance for a complex, broadband signal. Such an advance causes all frequency components and envelopes to appear to exit a circuit 0.078 ± 0.004 ms before entering it. Although this may seem counterintuitive, the results are not in conflict with relativistic causality and could lead to many useful devices. © *2004 American Institute of Physics*. [DOI: 10.1063/1.1773926]

It is a well-known consequence of Einstein's theory of special relativity that no "information" can travel faster than c, the speed of light in a vacuum.[1–3] However, it has also been shown that the group velocity, as measured by referencing the peaks of Gaussian-like envelope functions, can not only exceed c[4–14] but can even take on negative values in transparent media with optical gain,[9–11] bandpass amplifiers,[12,13] and coaxial photonic crystals.[14] Until now, all observations of superluminal propagation have been limited to simple Gaussian-like envelope functions. Here we report the superluminal time advance of a complex, broadband audio signal using a simple electronic circuit. Once the signal enters the circuit, all parts of the waveform, not just the peaks of envelopes, become uniformly time advanced and exit the circuit nearly a tenth of a millisecond before entering the circuit's input.

Any complex electromagnetic signal can be thought of as a superposition of multiple sinusoidal waveforms, each with a specific frequency and amplitude.[15] Upon entering any medium other than vacuum, these frequency components will experience different phase shifts and gains. This can lead to deformation of the signal and a reshaping of the envelope.[16] For such a signal, a minimum of two velocities must be defined (Sommerfeld and Brillouin[3] defined as many as five). The two most commonly used to describe the propagation of waveforms are the *group velocity*, $v_g = d\omega/dk$, and the *phase velocity*, $v_p = \omega/k$, where ω and k are the angular frequency and wave vector magnitude, respectively. The group velocity corresponds to the speed at which the peak of an envelope function propagates, while the phase velocity corresponds to the speed at which the peaks of the individual sinusoidal frequency components of the waveform propagate. When dealing with electronic circuits, it is more convenient to discuss *group delays*, $t_g = -d\varphi/d\omega$, and *phase delays*, $t_p = -\varphi/\omega$, where φ is the phase of an individual frequency component, since velocities are not easily defined for propagation within an electronic circuit (i.e., the distances over which signals travel are difficult to define). Thus, the group delay represents the time it takes the envelope of a function to traverse a given circuit and the phase delay is the time it takes a single frequency component to traverse the circuit. For the discussion that follows, we will use the terms *phase advance* and *phase delay* to refer to any forward and backward time shifts, respectively, and the term *phase* to refer to the angular advance of a single frequency component.

Previous experiments with electronic circuits have reported negative group delays for Gaussian envelope functions;[12,13] however, the amount of group delay varied depending on the center frequency of the wave packet. In order to uniformly advance a more complex, broadband signal, a constant phase advance is required over the entire signal bandwidth. By constructing a circuit that results in a constant phase advance and gain, one can achieve uniform superluminal time advancement of a complex signal. Figure 1 shows a double-bandpass amplifier used to achieve a constant phase advance with minimal gain outside the bandpass. By simple circuit analysis, the input and output voltages as a function of angular frequency for this circuit are related by:

$$\Delta\tilde{V}_{\mathrm{out}}(\omega) = T(\omega)\Delta\tilde{V}_{\mathrm{in}}(\omega), \quad (1)$$

where

$$T(\omega) = \left[1 + \left(\frac{1}{R2}\right)\left(\frac{1}{(R1 - \mathbf{i}\omega L1)^{-1} - \mathbf{i}\omega C1}\right)\right]\left[1 + \left(\frac{1}{R4}\right) \times \left(\frac{1}{(R3 - \mathbf{i}\omega L2)^{-1} - \mathbf{i}\omega C2}\right)\right], \quad (2)$$

is the transfer function with component values defined in Fig. 1 and $\mathbf{i} \equiv \sqrt{-1}$. The circuit's effect on transmission amplitude and phase for any input can be obtained from this transfer function, where the transmission amplitude is given by $|T(\omega)|$ and the phase is given by $\varphi(\omega) = \arg[T(\omega)]$.

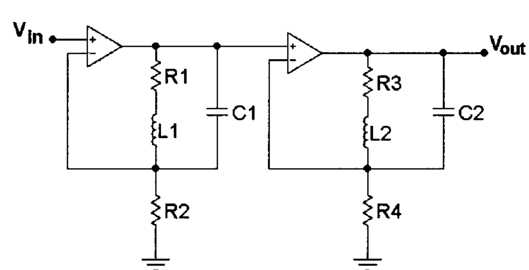

FIG. 1. Double bandpass amplifier. Component values are: R1=R3=39.4 Ω, R2=R4=23.8 kΩ, L1=1.5 H, L2=0.38 H, C1=0.897 nF, C2=1.0 nF. Operational amplifiers are Texas Instruments model TL072CP.

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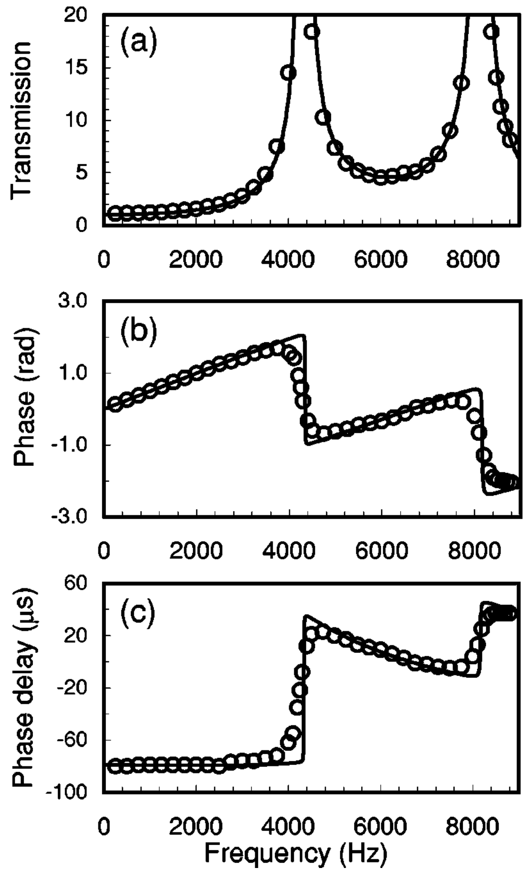

FIG. 2. Circuit properties: (a) transmission amplitude; (b) phase; (c) phase delay. Open circles correspond to experimentally measured values. Solid curves are obtained using the transfer function of Eq. (2).

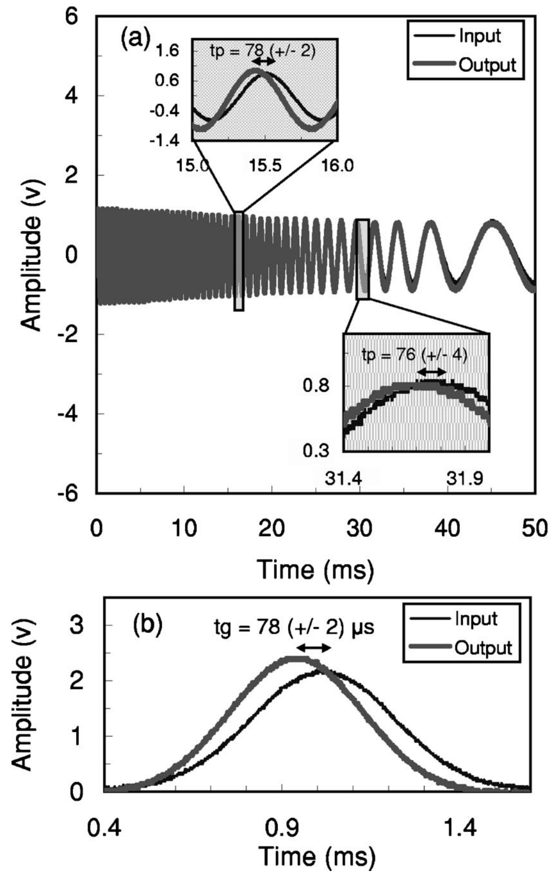

FIG. 3. Equivalence of group and phase delays in the double bandpass circuit. (a) Input and output voltages obtained from a frequency modulated sinusoid starting at 3 kHz and ending at 100 Hz. Insets show similar phase advances for different frequencies; (b) group advance for a base band envelope. Phase advances in (a) and the group advance in (b) are approximately equal in value.

Figure 2 shows the transmission amplitude, phase, and phase delay obtained for the double-bandpass circuit of Fig. 1. It can be seen from Fig. 2(c) that all frequencies in the range 0–3 kHz will be uniformly time advanced, since the phase delay is negative, by $\sim 80\ \mu$s and from Fig. 2(a) will experience very little amplification or attenuation. We also note that the phase, as shown in Fig. 2(b), has a positive, linear slope in the region from 0 to 3 kHz and that $\varphi(0)=0$. Thus, the group delay and phase delay are equal in this region:

$$t_g = -\frac{d\varphi}{d\omega} = -\frac{\varphi}{\omega} = t_p. \tag{3}$$

Previous experiments[12,13] with bandpass amplifiers have obtained phase versus frequency plots whose slopes were positive, corresponding to a negative group delay, but not linear. Such a situation leads to a region where both the group delay and phase delay are negative; however, the delays are not constant nor are the values equal. Thus, if a signal contains a spread of frequencies, different components of the signal will experience different group and phase delays so that the output signal may look very different from the input signal. Thus, only signals with simple, narrow bandwidth envelope functions can be time advanced without distortion in such a circuit.

The equivalence of group and phase delays for the double-bandpass amplifier shown in Fig. 1 can be seen explicitly in the experimental results of Fig. 3. Figure 3(a) shows the input and output signals for a frequency-modulated sinusoid starting at 3 kHz and ending at 100 Hz. The output experiences a phase advance of $\sim 78\ \mu$s at all frequencies. Figure 3(b) similarly shows the input and output signals for a base band envelope with frequency components from approximately dc to 3.5 kHz. The peak of the output envelope is measured to exit the circuit $78 \pm 2\ \mu$s before the input peak enters the circuit. The slight amplification of the output is a result of increased gain for the higher frequency components near the passband of the amplifier.

To take full advantage of this circuit, we now consider an arbitrarily complex signal containing frequencies from 0 to 3 kHz. Since the audible spectrum is $\sim$20 Hz–20 kHz, an electrical signal obtained from a frequency limited music sample will provide a sufficiently interesting case study. Figure 4 shows the input and output signals obtained using the double-bandpass circuit with an audio frequency input. The insets of the figure show the apparent exiting of the output signal prior to the input signal entering the circuit. The input and output signals were then attached to a speaker for comparison. With the exception of a slight amplification of the output due to the gain associated with the operational amplifiers, the two signals were nearly indistinguishable; however, in order for a human to detect the temporal advance audibly, a time difference on the order of 30 ms is needed.

Here we note that this superluminal time advance is very different from those observed in previous studies.[4–14] In those studies, it was the advance of the envelope function of a Gaussian wave packet that exhibited superluminal behav-

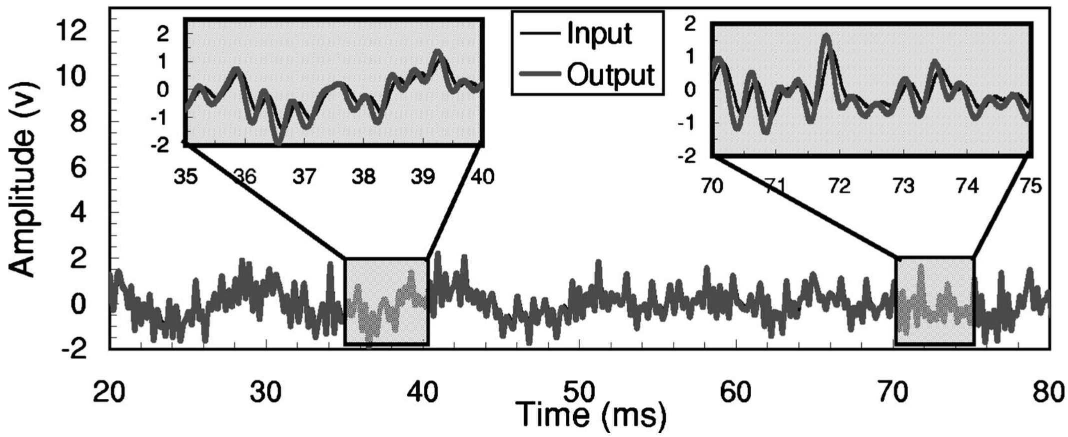

FIG. 4. Time advance of a complex signal. The output of a broadband signal containing frequency components from 20 Hz to 3 kHz appears to exit the circuit prior to its input entering the circuit. There is a slight amplification of the higher frequency components due to the bandpasses near 4.3 and 8.2 kHz.

ior. If the speed of the carrier frequency were different from the speed of the envelope, then the exiting waveform would be distorted with respect to the input waveform. In such cases it is necessary to distinguish between the phase and group velocities. Furthermore, the regions of constant superluminal group velocities or phase velocities have been limited to narrow frequency bandwidths, and the incident wave packets had to be similarly limited.

The frequency range over which superluminal time advances can be achieved in the double-bandpass filter can be extended by moving the passbands to higher frequencies. We constructed an appropriately modified circuit to achieve time advances for frequencies up to 20 kHz, but the amount of time advancement was reduced. Typically, each bandpass contributes a maximum advancement of $\varphi=\pi/2$, occurring for frequencies just below the bandpass peak. The amount of advance can be increased by the addition of multiple bandpasses; however, the additional gain stages will increase the transmission amplitude for the higher frequency components more than for the lower frequency components leading to amplitude distortion. It was also found that the bandpass maxima needed to be offset by several kilohertz in order to avoid ringing due to the high gain associated with each bandpass.

Finally, we note that these results are not in conflict with special relativity or the principle of causality,[1–3] which state that no *information* can travel faster than c. The most commonly agreed upon definition of information when discussing causality is that of an abrupt discontinuity.[12–17] Such a discontinuity must, by Fourier's theorem,[15] contain an infinite frequency spectrum. To achieve constant phase and group advances, our system requires an input whose bandwidth is completely contained within the region of constant advance (0–3 kHz in our case). Since the discontinuity must contain frequencies beyond this range, this type of signal cannot be uniformly time advanced. Thus, there is no causality violation given these definitions.

We have shown a uniform advance of all frequency components from a bandwidth limited audio signal. The maximum extent to which these signals can be advanced and the limitations on the types of signals that can be used are currently being studied. It is hoped that such studies will lead to the development of many useful applications for increasing signal transmission rates in everyday devices.

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