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Slow electromagnetic pulse propagation through a narrow transmission band in a coaxial photonic crystal

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We demonstrate the slow group-velocity propagation of electromagnetic pulses through a narrow transmission band of a simple coaxial photonic crystal. The narrow transmission band was formed by creating a defect in a periodic coaxial cable filter which resulted in a narrow-frequency passband within an otherwise forbidden band stop region. Strong normal dispersion within this region causes the group velocity of the pulse to be slowed well below the speed of light and below the expected propagation speed in a coaxial cable. This phenomenon is essentially similar to the much-studied slow light propagation observed in atomic vapors through regions of strong dispersion. The simplicity with which this phenomenon can be observed in a one-dimensional coaxial photonic crystal makes this a versatile experiment for such studies. Group velocities of $0.30c$, where c is the speed of light in a vacuum, were observed. © *2003 American Institute of Physics.*
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Much recent attention has been given to the topic of slow, or even stopped, group velocities of optical pulses in atomic vapors as a result of highly dispersive, narrow transmission regions within the vapors.[1–6] Typically these experiments employ the phenomenon of electromagnetically induced transparency[7] to create a narrow transmission band within an otherwise optically opaque medium. This narrow transmission band leads to a highly dispersive region in which slow group velocity pulse propagation is possible. It has been theorized that a similar effect should be possible in the region of a defect mode within a photonic band gap material.[8] The origin of this effect can be understood in optical terms from the group velocity, v_g, of a pulse given by

$$v_g=\frac{d\omega}{dk}=\frac{c}{n+\omega\left(\frac{dn}{d\omega}\right)}, \tag{1}$$

where n is the refractive index. Regions of strong normal dispersion $(dn/d\omega>0)$ can give rise to very slow group velocities whereas regions of strong anomalous dispersion $(dn/d\omega<0)$ can give rise to the well-described superluminal effects.[9–14]

Herein we describe a simple experimental configuration that leads to slow-group-velocity electromagnetic pulse propagation through a narrow transmission band in a one-dimensional coaxial photonic crystal. This effect has been experimentally observed in a periodically structured electromagnetic wave guide in the presence of an induced defect mode. The experiment is simple and inexpensive both in terms of the fabrication of the coaxial photonic crystal and in the instruments needed to determine the slowed group velocity.

The coaxial photonic crystal used for this study consisted of alternating sections of coaxial cable (RG58 and RG62), which have differing impedances (50 and 93 Ω, respectively). The structure was assembled from 13 alternating sections of cable with RG58 at both ends. The lengths of the cables were chosen to form a quarter wavelength stack leading to a band gap at 8.0 MHz. Due to the differing velocity factors of these cables ($0.66c$ in RG58 and $0.85c$ in RG62, where c is the speed of light in vacuum), the cables were cut to different lengths (6.19 and 7.97 m, respectively) to achieve the desired periodicity. Next, we wanted to induce a defect within the periodicity of the wave guide. In order to do this, the length of the middle coaxial cable element within the array was doubled to produce a narrow transmission band within the center of the band gap. The total lengths for the coaxial photonic crystals with and without a defect were 97.34 and 91.15 m, respectively.

In order to determine the frequency extent of the forbidden band gap and of the defect mode in the two coaxial photonic crystals, an impulse response experiment was used to measure the transmission spectra for each of the waveguides. A short electrical pulse was created by filtering a square wave using a simple high pass RC filter ($R=820\ \Omega$ and $C=68$ pF). The pulse contained a broad range of frequencies extending beyond 15 MHz. The time domain data of the pulse was then recorded after having traversed the waveguide. The Fourier transform of this time domain data was divided by that of the pulse transmitted through a single cable to obtain the transmission function for the waveguides. Figure 1 shows the transmission function for the coaxial photonic crystals with and without the defect mode.

In order to determine the group velocity through the band gap region as a function of frequency, we measured the transit times for the peaks of the narrow bandwidth pulses within the coaxial photonic crystal. Narrow bandwidth pulses were created by using one function generator to modulate the output of the second function generator. The experimental configuration is shown schematically in Fig. 2.

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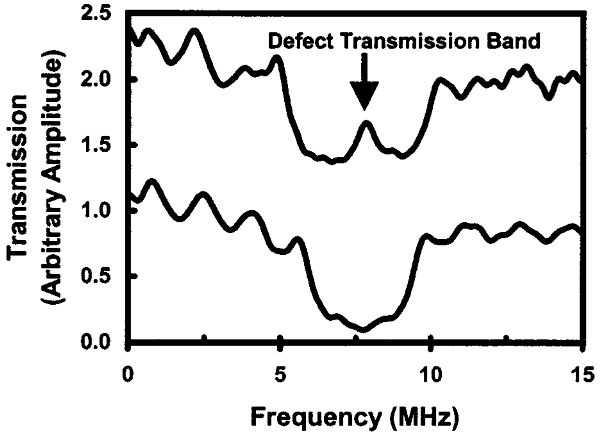

FIG. 1. Transmission spectra for the 91.15 m coaxial photonic crystal (bottom) and the 97.34 m coaxial photonic crystal with a central defect mode (top). Both the band gap and defect mode are clearly visible. The transmission function of the upper trace has been offset for clarity.

The first signal generator (BK Precision 30011B) was used to create a low frequency (70 kHz) sinusoidal envelope that would then modulate a higher frequency (2–15 MHz) sinusoidal signal produced by a second signal generator (BK Precision 4040A). The result is a train of sinusoidally modulated pulses with carrier frequencies in the megahertz regime. The width of the pulse is set by the first signal generator, which creates the envelope. The second signal generator can then independently adjust the carrier frequency. Upon transmission through the coaxial photonic crystal, the pulses are recorded at the oscilloscope (Agilent 54622D). Due to the close spatial proximity of adjacent pulses, a $50\ \Omega$ termination resistor was attached at the oscilloscope's input to reduce back reflections resulting from an impedance mismatch at the oscilloscope. With knowledge of the distance traveled by a pulse through the waveguide and the corresponding transit time, we then determined the group velocities. Such measurements were made for several pulses with carrier frequencies between 2 and 15 MHz. One advantage of this experimental setup is that the carrier frequency can be adjusted in real time, and the electrical pulses can be seen to be slowing down by the observation that they move to later times on the oscilloscope's display.

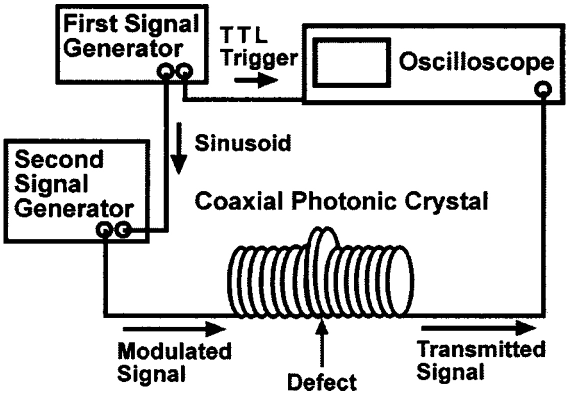

FIG. 2. Experimental configuration for measuring the group velocity of pulse propagation through coaxial photonic crystals. The output of signal generator 1 is used to amplitude modulate the carrier frequency produced within signal generator 2.

To accurately determine the transit time for the electrical pulses, we made several considerations. It was found that the peak of the sinusoid emitted from the first signal generator coincided with the positively rising edge of the transistor–transistor logic (TTL) out signal. With this in mind, two cables of identical length were used to ensure that the peak of the sinusoid reached the input of the second signal generator as the positively rising edge of the TTL signal triggered the oscilloscope. The TTL signal would then be kept on the oscilloscope's display to represent the arrival of the peak of the pulse at the second signal generator.

It was then necessary to determine the processing time needed by the second signal generator in order to produce the modulated signal. First, we allowed the sinusoid produced by the first signal generator to travel through two cables (connected in series) of equal length and of equal impedance before reaching the oscilloscope. The transit time of the peak of the sinusoid was noted. Next, we inserted the second signal generator into the sinusoid's path between the two cables. We compared the transit time of the peak of this amplitude-modulated signal to the previous transit time for the peak of the sinusoid alone. The timing difference is due to the processing delay of the second signal generator and had a value of 100 ± 10 ns.

The transit time for the pulse to traverse the coaxial photonic crystal is determined by taking the time difference between the peak of the received pulse, transmitted through the coaxial photonic crystal, and the rising edge of the TTL signal and subtracting the processing time at the second signal generator. Because 70 000 pulses pass through the structure each second, it is possible to visually locate an average peak position of the pulse on the oscilloscope's display. We were then able to adjust the carrier frequency and watch the pulses advance and slow down in real time as the carrier frequency enters the band gap region and reaches the defect mode. Figure 3(a) shows the transmission spectrum of the coaxial photonic crystal and Fig. 3(b) shows group velocity measurements made for several carrier frequencies ranging from 3 to 12 MHz. Superluminal as well as negative group velocities were observed in the band gap region in agreement with previous experiments[9] and slow pulse propagation was observed in the region of the defect mode. Figure 4 depicts three pulses with different carrier frequencies, 4.0, 7.8, and 9.3 MHz, corresponding to different group velocities, $0.74c$, $0.30c$, and $2.3c$, respectively. The pulse amplitudes have been adjusted to be approximately equal in size.

The asymmetrical group velocity distribution about the defect mode is thought to arise as a result of minor flaws and intrinsic losses within the waveguide. Such disturbances can cause ripples in the transmission spectrum within the band gap and defect mode resulting from perturbed interference conditions. One such flaw can be seen on the high frequency edge of the defect mode in Fig. 3 which could explain why slow group velocity is more predominant on the lower frequency region of the defect. There is also a region of relatively slow group velocities around 10.6 MHz resulting from band edge effects.[15]

The slowing in group velocity is not as dramatic in this experiment as in slow optical pulses in atomic vapors because the transmission band associated with the defect mode is not particularly narrow. Making the coaxial photonic crys-

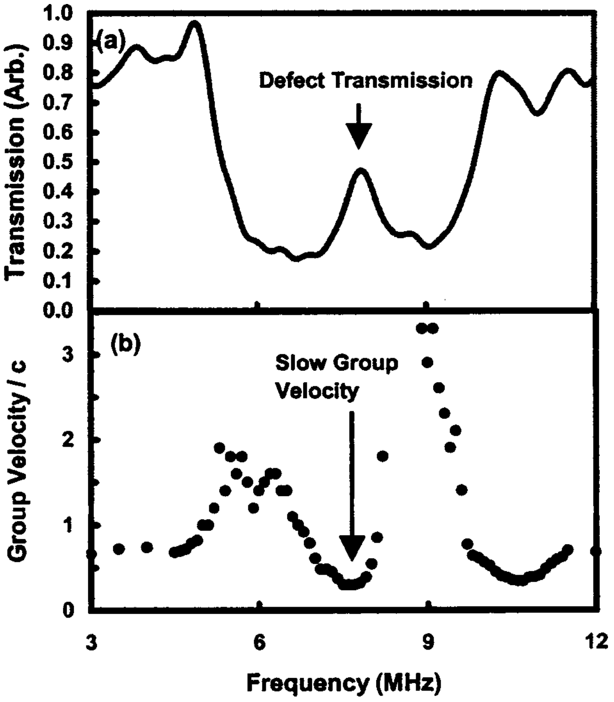

FIG. 3. (a) Transmission spectrum of the coaxial photonic crystal with a central defect (b) the group velocity of pulses with carrier frequencies ranging from 3 to 12 MHz. Slow group velocity propagation is observed in the region of the defect mode.

tal longer can in principle decrease the width of the defect mode. However, the intrinsic dielectric loss of the coaxial cable prevents the creation of a very narrow transmission band.

In conclusion, we have demonstrated the slow group velocity pulse propagation of an electromagnetic pulse in a highly dispersive region corresponding to a narrow passband within a coaxial photonic crystal. Although experiments have been performed using atomic vapors to slow optical pulses through narrow transmission bands in highly dispersive regions, we now experimentally slow group velocity propagation mediated by the defect state in a photonic band gap filter. Furthermore, the simplicity of the experimental setup makes this an appealing method for studying group velocity manipulation in periodic systems.

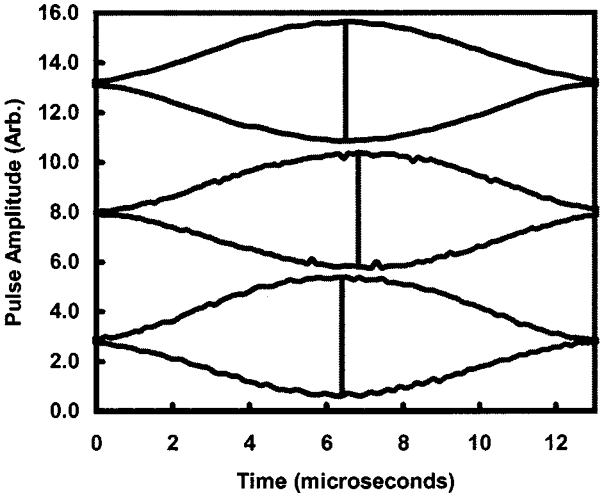

FIG. 4. Three pulses with carrier frequencies of 4.0, 7.8, and 9.3 MHz (top to bottom) corresponding to different group velocities, 0.74c (normal propagation), 0.30c (slow propagation), and 2.3c (superluminal propagation), respectively. The plot shows only the pulse envelope not the high carrier-frequency oscillations. A vertical line in each envelope marks the position of the pulse peak. Pulse amplitudes have been adjusted to be approximately equal in size.

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