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Negative group velocity pulse tunneling through a coaxial photonic crystal

J. N. Munday and W. M. Robertson[a)]
Department of Physics and Astronomy, Middle Tennessee State University, Murfreesboro, Tennessee 37132

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An easily configurable experimental system is described in which negative group velocity tunneling of electrical pulses can be studied. Negative group velocity corresponds to the case in which the peak of a tunneled pulse exits before the peak of the incident pulse has entered the sample. In the experiments outlined herein, the tunneling occurs through the forbidden transmission region of a quarter-wavelength interference filter created from alternating segments of two different impedance coaxial cables. The equipment used for these experiments consists of two function generators and an oscilloscope, items typically found in most science departments. © *2002 American Institute of Physics.* [DOI: 10.1063/1.1508172]

In this letter, we describe a simple experimental configuration to demonstrate the tunneling of electrical pulses with group velocities, v_g, whose values exceed infinity—so-called negative group velocity tunneling. Such a circumstance occurs when the peak of a pulse tunneling through an attenuating sample, in our case a coaxial photonic crystal, exits before the peak of the input pulse has reached the beginning of the sample. Although this effect is highly counterintuitive, it is a natural extension of the recently much studied phenomenon of pulse tunneling which has established, in a myriad of different experimental configurations,[1–3] that group velocities can exceed the speed of light in a vacuum, c.

Many authors have previously pointed out that negative group velocities are possible[4,5] particularly in systems with gain; however, there have been few clear experimental demonstrations of this phenomenon in passive systems. One of the earliest experiments to measure the group velocity of optical pulses tunneling through an absorbing medium demonstrated v_g values that were negative;[6] however, because of the autocorrelation method employed to measure the pulse velocity, the results were initially contentious.[7] Conversely, a more recent experiment[8] with electrical pulses traveling through the forbidden transmission region of a bandstop filter clearly demonstrated negative group delays. However, for a filter, the concept of a group velocity is not defined.

In the work described herein, we demonstrate, in a simple, unequivocal experiment, a negative group velocity for pulse tunneling through a coaxial photonic crystal—an interference filter made of alternating quarter-wavelength segments of two different impedance coaxial cables. The basic experiment is similar to that described recently by Haché and Poirier[9] in which they observed v_g values up to three times the speed of light in a vacuum. The prime difference in our experiment is the use of a coaxial photonic crystal with a higher impedance mismatch that permits access to larger group delays. Furthermore, the experimental equipment used is much simpler, an oscilloscope and two function generators (one with an AM modulation facility), items typically available in most science departments.

The experimental configuration used for measuring the group velocity of pulses is shown schematically in Fig. 1. The first-signal generator (BK Precision 30011B) was used to produce a sinusoidal signal at 110 kHz to amplitude modulate the much higher frequency (5–15 MHz) output of the second-signal generator. The TTL synch out signal of the first-signal generator served as a trigger for the oscilloscope (Agilent 54622D). The output of the second-signal generator (BK Precision 4040A), when 100% modulated by the output of the first generator, produced a train of sinusoidal envelope pulses with carrier frequencies in the MHz frequency regime. The repetition rate of these pulses is set by the first-signal generator, which creates the envelope. The second-signal generator sets the carrier frequency that can be independently adjusted. These pulses then traverse the coaxial photonic crystal and are recorded at the oscilloscope. A 50 Ω termination resistor was attached at the input of the oscilloscope to eliminate back reflections resulting from an impedance mismatch at the input.

The coaxial photonic crystal used for this study consisted of alternating sections of two different types of coaxial cable, RG58 and RG62, with impedances of 50 Ω and 93 Ω, respectively. The structure was then assembled from 17 alternating sections of cable with RG58 at both ends. To create

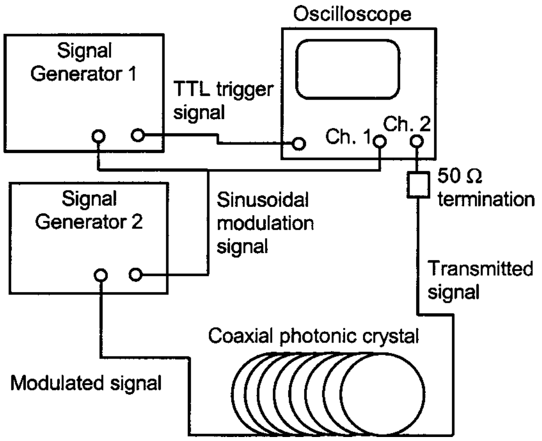

FIG. 1. Experimental arrangement for demonstrating negative group velocity tunneling. The output of signal generator 2 is amplitude modulated by the output of signal generator 1.

[a)]Electronic mail: wmr@physics.mtsu.edu

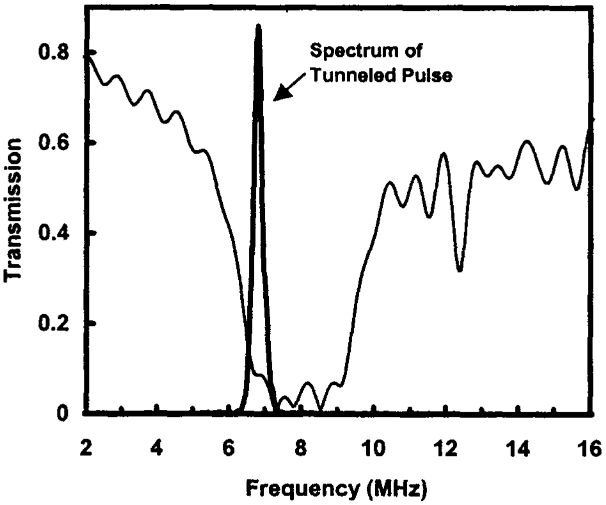

FIG. 2. Experimentally determined transmission function of the photonic band gap crystal (thin line). The narrow peak (thick line) overset on the transmission function corresponds to the frequency components of the tunneled pulse at 6.8 MHz showing that all of its frequency components lie within the forbidden band gap.

a forbidden transmission region, the cable lengths were cut to be a quarter-wavelength long at our chosen target band gap frequency of 8 MHz. Because of the different velocity factors in these cables (0.66*c* in RG58 and 0.85*c* in RG62, where *c* is the speed of light in a vacuum), the cables were cut to different lengths (6.19 m and 7.97 m respectively) to achieve the desired periodicity. The total length of the coaxial crystal was 119.5 m.

An impulse response experiment was performed to verify the existence and extent of the coaxial crystal band gap. A short pulse was created by high pass filtering the square wave output of one of the signal generators. The filter, created from a resistor and capacitor ($R = 820\,\Omega$, $C = 68\,\mathrm{pF}$), converted the rising and falling edges of the square wave into a pulse with a broad range of frequency components extending beyond 16 MHz. The pulse transmission through the coaxial photonic crystal was digitized and stored using the oscilloscope as was the time response for the pulse to travel through a straight piece of a 50 Ω cable. The Fourier spectra of the two time responses were divided to arrive at the transmission profile shown in Fig. 2. Figure 2 shows a clear band gap centered at 8 MHz and extending from 6 to 10 MHz.

In order to measure the group velocity of pulses traveling through the photonic coaxial crystal, we determined the arrival time of the peak of pulses both for carrier frequencies within the forbidden band gap region, where tunneling should occur, and in the transmitting region where we would expect normal propagation. The transition from outside the gap to inside the gap could be observed in real time on the oscilloscope by adjusting the carrier frequency on the second-signal generator. Even before doing careful measurements of the group velocity, it is clear that the system gives rise to negative group delays (and hence, negative group velocities) because the peak of the pulse traveling through the coaxial crystal moves to earlier times as the carrier frequency is adjusted from below the band gap to being within the band gap. As the carrier frequency is changed to values that lie within the band gap, there is a decrease in the pulse amplitude as expected. Eventually, the peak of the tunneling pulse occurs at a time that precedes the peak of the envelope pulse as monitored on the second channel of the oscilloscope. The effect is even more dramatic than it appears on the oscilloscope because the envelope signal from the first generator has traveled over a short (~2 m) cable to the oscilloscope whereas the tunneled pulse has traveled almost 120 m through the coaxial photonic crystal.

To accurately determine the transit time for these electrical pulses, several considerations must be made. The peak of the sinusoid emitted from the first-signal generator was determined to coincide with the positive rising edge of the TTL signal. With this in mind, two cables of identical length were used to ensure that the peak of the sinusoid reached the input of the second-signal generator as the positively rising edge of the TTL signal triggered the oscilloscope.

Next, we had to consider the processing time needed by the second-signal generator in order to produce the modulated signal. To do this, we allowed the sinusoid produced by the first-signal generator to travel through two cables (connected in series) of equal length and impedance before reaching the oscilloscope. The timing difference between the TTL signal and the peak of the sinusoid was noted. We then inserted the second-signal generator into the path of the sinusoid between the two cables. The timing difference between the TTL signal and the peak of this amplitude-modulated signal was then compared to the previous timing difference. The change in this timing difference is due to the processing delay introduced by the second-signal generator and had a value of $0.10 \pm 0.01\ \mu\mathrm{s}$.

To determine the time spent by the pulse within the coaxial photonic crystal, two measurements were made. First, the position of the peak of the sinusoid entering the second-signal generator was read from the oscilloscope. (This also corresponds to the rising edge of the TTL trigger, which is on the display.) Second, the position of the peak of the pulse after having traversed the coaxial photonic crystal was recorded. Because 110 000 pulses pass through the structure each second, it is possible to visually find an average peak position on the oscilloscope. The transit time is then simply the timing difference between the peak of the entering sinusoid and the peak of the recorded pulse minus the processing time at the second-signal generator. We were then able to adjust the carrier frequency and watch the pulses advance in real time as the carrier frequency entered the band gap. A typical trace from three such pulses entering the band gap region is shown in Fig. 3. The top pulse in Fig. 3 had a carrier frequency of 6.3 MHz and a corresponding group velocity of $.77c$. The second pulse had a carrier frequency of 6.5 MHz and became superluminal having a group velocity of $4.0c$. The third pulse had a carrier frequency of 6.8 MHz and a corresponding group velocity of $-1.2c$ (i.e., the peak of the tunneled pulse arrived $0.32 \pm 0.02\ \mu\mathrm{s}$ prior to the peak of the input pulse entering the crystal). It became difficult to accurately determine peak values for the higher-frequency pulses due to attenuation and deformation of the pulse within the band gap region.

From the theoretical viewpoint, the observation of a negative group velocity in this system is somewhat surprising. The authors of Ref. 10 have shown that, for a loss-less

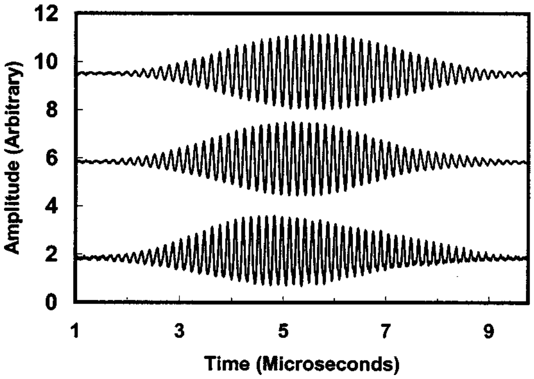

FIG. 3. Three tunneled pulse traces at 6.3 MHz, 6.5 MHz, and 6.8 MHz (top to bottom) showing that the tunneled pulse moves to earlier times as it moves into the forbidden transmission region. The pulse amplitudes have been adjusted to be roughly equal in size.

one-dimensional filter, the group velocity has an upper limit of $c/|t|^2$ where t is the amplitude transmission of the filter. In this loss-less model, negative group velocities are not possible. The experimental system studied here does exhibit significant loss, as is evident in Fig. 2 from the reduced transmission at frequencies outside the band gap. It is possible that the addition of loss might account for the observed negative group velocity. Conversely, the effect might arise due to phase effects associated with structure evident within the forbidden transmission region. This structure is due to Fabry–Perot-type interference between the ends of the entire photonic crystal.

In conclusion, we have demonstrated a simple experimental configuration that exhibits negative group velocity. Although this effect is well acknowledged as a logical extension of the other superluminal tunneling effects, it engenders more skepticism because the tunneling pulse *appears* to exhibit some prescience in knowing the shape of the incident pulse even before it has arrived. The experimental configuration described here should make this elusive effect amenable to more widespread study.

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