Review

Tao Gong, Matthew R. Corrado, Ahmed R. Mahbub, Calum Shelden and Jeremy N. Munday*

Recent progress in engineering the Casimir effect – applications to nanophotonics, nanomechanics, and chemistry

https://doi.org/10.1515/nanoph-2020-0425
Received July 24, 2020; accepted August 26, 2020; published online September 24, 2020

Abstract: Quantum optics combines classical electrodynamics with quantum mechanics to describe how light interacts with material on the nanoscale, and many of the tricks and techniques used in nanophotonics can be extended to this quantum realm. Specifically, quantum vacuum fluctuations of electromagnetic fields experience boundary conditions that can be tailored by the nanoscopic geometry and dielectric properties of the involved materials. These quantum fluctuations give rise to a plethora of phenomena ranging from spontaneous emission to the Casimir effect, which can all be controlled and manipulated by changing the boundary conditions for the fields. Here, we focus on several recent developments in modifying the Casimir effect and related phenomena, including the generation of torques and repulsive forces, creation of photons from vacuum, modified chemistry, and engineered material functionality, as well as future directions and applications for nanotechnology.

Keywords: Casimir effect; Casimir force; Casimir torque; quantum electrodynamics; quantum fluctuations.

***Corresponding author: Jeremy N. Munday**, Department of Electrical and Computer Engineering, University of California, Davis, CA 95616, USA, E-mail: jnmunday@ucdavis.edu. https://orcid.org/0000-0002-0881-9876
Tao Gong, Department of Electrical and Computer Engineering, University of California, Davis, CA 95616, USA; Department of Materials Science and Engineering, University of California, Davis, CA 95616, USA. https://orcid.org/0000-0002-5033-8750
Matthew R. Corrado, Department of Physics, University of California, Davis, CA 95616, USA. https://orcid.org/0000-0002-3141-7322
Ahmed R. Mahbub and Calum Shelden, Department of Electrical and Computer Engineering, University of California, Davis, CA 95616, USA. https://orcid.org/0000-0002-9987-9777 (A.R. Mahbub). https://orcid.org/0000-0001-9404-4247 (C. Shelden)

1 Introduction

Quantum fluctuations of electromagnetic waves are a fascinating consequence of the quantization process that occurs when light is pushed to the quantum regime. These fluctuations are responsible for many everyday phenomena from spontaneous emission and the Lamb shift to the wetting processes at solid–liquid interfaces and the biological grip of geckos [1–4]. Another intriguing phenomenon that results from the confinement of quantum fluctuations is the Casimir effect, which occurs when two or more materials are placed within close proximity and perturb the available vacuum modes. If the vacuum energy density varies with displacement of one of the bodies with respect to the others, a force results (Figure 1). In 1948, Casimir [5] considered how the density of vacuum modes changed when two parallel, uncharged metal plates were brought nearby, and he found an attractive force exists between them, which is given by $F = -\frac{\pi^2 \hbar c}{240 d^4}$, where d is the separation between the plates and $\hbar$ and c are fundamental constants. In the decades that followed, several experiments confirmed Casimir's prediction [6–10], and methods have been developed to improve the measurement by taking into account surface roughness [11–14], electrostatic patch potentials [15–21], and thermal contributions [22, 23]. Recently, experiments have begun to show other novel regimes where the strong deviations in the magnitude of the force or power law can be obtained through modification of the geometry [24–31] or optical properties [32–39].

Our ability to tailor electromagnetic boundary conditions is opening new opportunities to engineer not only the Casimir force but also other related phenomena ranging from the generation of photons from moving cavities to modifications of chemical reactions and material phase transitions. In this review, we will highlight some of the recent advancements in the field of Casimir physics and point to some of the exciting new developments and future directions. Given the number of excellent review articles and books pertaining to the Casimir effect [1–3, 40–51], we will limit our focus to connecting recent works with new

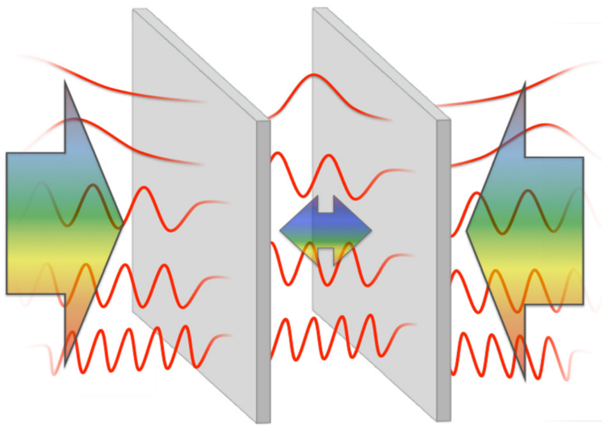

Figure 1: Casimir effect. Quantum fluctuations of electromagnetic fields give rise to a virtual photon pressure on two metal plates. The pressure between the plates is smaller than the pressure from outside, leading to an attractive force between them.

directions for engineering the Casimir effect and related phenomena.

This review article is divided as follows. After a brief introduction to the Casimir force and an historical perspective on its measurement, we discuss how both the optical properties and the geometry of the objects involved affect the magnitude and sign of the force. Next, we describe how optically anisotropic materials can give rise to a new rotational effect, known as the Casimir torque. We follow with a discussion of how systems can be pushed out of equilibrium to further modify the force. Beyond the generation of forces and torques, we discuss how nonadiabatic disturbances of the vacuum field state can result in the generation of photons through the rapid modulation of an electromagnetic boundary and how the Casimir energy can be used to modify molecular states and chemistry. Finally, we conclude with a discussion of applications to nanoscale mechanical devices and other future directions.

Within a decade of Casimir's prediction, initial experiments by Abrikosova and Derjaguin [6] and Sparnaay [7] showed evidence of a force, but limitations in their ability to rule out spurious effects resulted in Sparnaay's claim that his measurements merely "do not contradict Casimir's theoretical prediction." Twenty years passed until van Blokland and Overbeek [8] presented clear evidence of the Casimir force using a chromium layer and compensating voltage to reduce electrostatic interactions. For the next two decades, there were no significant experimental advances until the precision measurement of the Casimir effect by Lamoreaux [9] using modern experimental techniques. This measurement ushered in a new wave of Casimir force experiments using a variety of techniques [10, 23, 24, 35, 52–57]. Figure 2 gives a brief overview of several experimental achievements pertaining to detecting and modifying the Casimir force with a focus on early experimental techniques and advancements [5–10, 55], the effect of the material's optical properties [27, 32, 33, 35–38, 58–60], and the effect of surface texturing and geometry [24, 26, 28–30, 61].

2 Effect of dielectric response

While Casimir's derivation invoked perfect electrical conductors (i.e., ideal mirrors) for the two plates, the force persists when the materials are described by realistic optical properties [62], and its strength depends on the reflectivity of these interacting surfaces (higher reflectivity results in a stronger force). Most metals are excellent broadband reflectors; hence, the interaction between metallic plates is usually much stronger than that between dielectrics. For example, the Casimir force was found to decrease by a factor of two when one of the metallic surfaces was replaced with a transparent conducting oxide [36]. Similarly, there is interest in observing the Casimir effect as a material goes through a phase change, which modifies its optical and electrical properties [60, 63, 64]. However, just because a surface is transparent in the visible region, it does not mean that a significant reduction of the Casimir effect will occur because the force is extremely broadband (see, for example, the case of hydrogen-switchable mirrors, where no significant difference was found between the transparent and reflective states [33]). Further, the individual frequency contributions to the force are highly oscillatory, containing both attractive and repulsive components [65, 66], which adds complexity to both the calculation (e.g., ensuring that the total force converges numerically) and to using intuition from optics to engineer the interaction (e.g., metamaterials involving resonant structures may appear to greatly affect the force [67, 68], but in reality, the results are usually minor [69–71]).

To efficiently calculate the Casimir force between real materials, we often perform a contour integration in the complex plane (Wick rotation) to remove the oscillatory behavior that occurs for real frequencies. This procedure amounts to replacing ω with $i\xi$ (noting that no poles exist in the upper half of the complex plane as a physical consequence of causality) and causes the force integrand to decay rapidly with increasing ξ [62, 66, 72]. Another consequence of the Wick rotation is that the resulting dielectric response $\epsilon(i\xi)$ is a monotonically decreasing function with respect to $i\xi$, which is why the mathematical integration to obtain the force is greatly simplified. From a physical perspective, $\epsilon(i\xi)$ depicts how the charges within a material respond to exponentially increasing fields rather than oscillating ones [73].

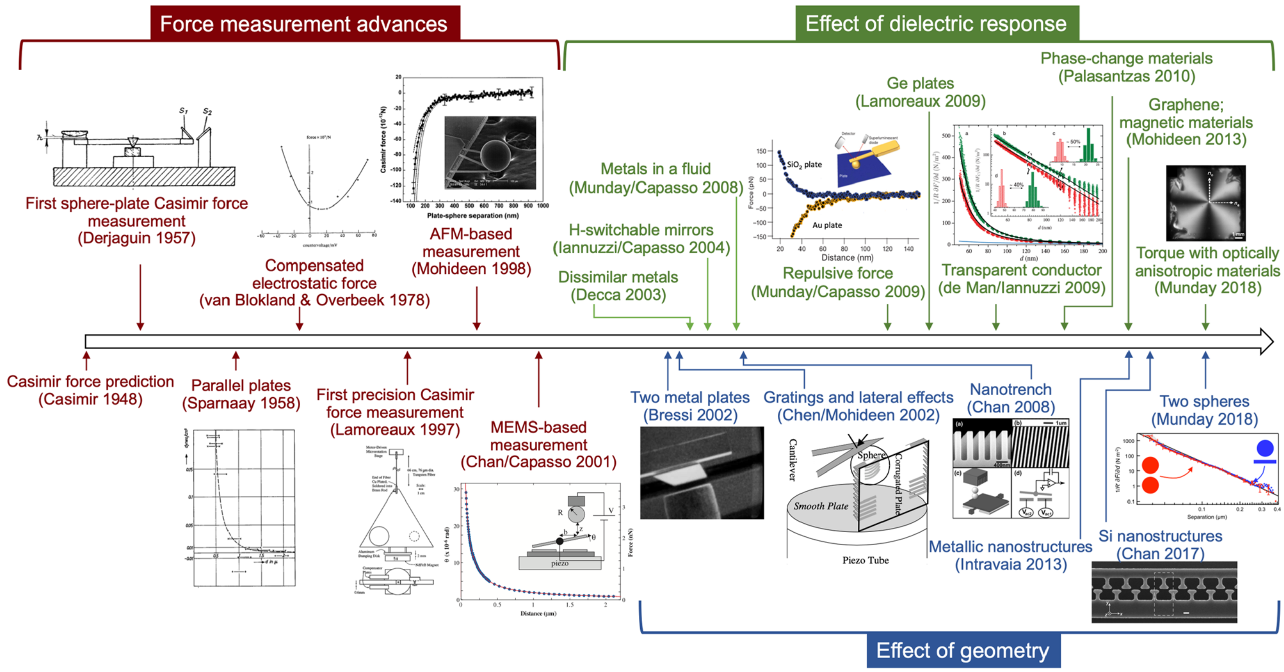

Figure 2: A brief history of Casimir force measurements and the use of dielectric response and geometry to modify the interaction. The images in this chart are adapted with permission from the studies by Abrikosova and Derjaguin [6]. Copyright 1957, MAIK Nauka/Interperiodica. Sparnaay [7]. Copyright 1958, Elsevier B.V. van Blokland and Overbeek [8]. Copyright 1978, Royal Society of Chemistry. Lamoreaux [9]. Copyright 1997, American Physical Society. Mohideen and Roy [10]. Copyright 1998, American Physical Society. Chan et al. [55]. Copyright 2001, The American Association for the Advancement of Science. Munday et al. [59]. Copyright 2009, Nature Publishing Group. de Man et al. [36]. Copyright 2009, American Physical Society. Somers et al. [32]. Copyright 2018, Nature Publishing Group. Bressi et al. [24]. Copyright 2002, American Physical Society. Chen et al. [29]. Copyright 2002, American Physical Society. Chan et al. [61]. Copyright 2008, American Physical Society. Tang et al. [30]. Copyright 2017, Nature Publishing Group. Garrett et al. [26]. Copyright 2018, American Physical Society.

For two mirror symmetric objects made of the same material, the force is always attractive, irrespective of the intermediate medium [74]; however, if the symmetry is broken, a net repulsive force can arise, provided the interacting materials are properly chosen. The condition to achieve repulsion between two semi-infinite half-spaces separated by a third material is $\epsilon_1(i\xi) > \epsilon_3(i\xi) > \epsilon_2(i\xi)$, where the subscript 1, 2, and 3 denote the materials of interacting objects (1 and 2) and intervening medium (3) [75]. It should be noted that this condition has to hold over a sufficiently large frequency range to render the force repulsive; hence, finding materials that satisfy this condition is not easy. In 2009, the first measurement of the long-range repulsive Casimir force was reported using a gold sphere, a silica plate, and a fluid of bromobenzene [59], which suggests a potential way to mitigate stiction problems in microelectromechanical systems (MEMS) or nanoelectromechanical systems (NEMS), and also suggests the possibility of other intriguing phenomena, such as quantum levitation, superlubricity [76], and stable mechanical suspension of objects in fluids [77]. Recently, a teflon-coated gold surface was also used to stably trap a gold nanoplate in ethanol [78], and new theoretical approaches are being developed to analyze other geometries in a fluid [79]. For optically anisotropic materials, it has been pointed out that two identical birefringent materials can lead to repulsion for particular orientations of the objects, provided the optical properties are specifically chosen [80]. Materials with a strong magnetic response have also been proposed to yield a repulsive force when placed near another material with a strong electric response [81, 82]; however, questions have been raised about the ability to tailor such materials [83, 84]. One potential implementation involves superparamagnetic metamaterials [85], although the magnitude of the force appears to be beyond current experimental capabilities. Finally, materials that have $\epsilon(\omega) = 0$ can be used to create cavities that suppress quantum fluctuations over a particular bandwidth [86], which may provide an additional opportunity for modifying the Casimir effect if materials can be engineered over appropriate bandwidths.

These examples show how selecting and engineering the dielectric response over a broad frequency range can lead to significant modifications of the Casimir effect with potentially important technological applications for nanomechanical devices.

3 Effect of geometry

While Casimir's original derivation considered the force between two parallel plates, many alternative geometries have been explored both due to experimental convenience and for the possibility of generating repulsive forces. Experimentally, the plate–plate configuration proved challenging, as it is difficult to maintain parallelism between two plates at submicron separations as a result of the strongly nonlinear force power law (d^{-4}) that would amplify any tilt, leading to large variations in the force magnitude across the plate. For this reason, nearly all experiments have been performed in the sphere–plate configuration, where the Casimir force persists, albeit with a slightly different form: $F_C = -\frac{\pi^3 \hbar c R}{360 d^3}$, where R is the radius of the sphere (valid for $R >> d$).

New experimental setups are enabling researchers to now move beyond the sphere–plate configuration (see Figure 2). The first significant modification was achieved by corrugating one of the surfaces [29]. This configuration allowed for both a modification of the force as well as the observation of a lateral contribution. Advances in silicon microfabrication have also allowed for the use of MEMS devices with interlaced parts (similar to the interlaced comb-drive actuator) to explore more complex, in-plane geometries [30]. Atomic force microscope (AFM)-based techniques have been further expanded to allow for 3D alignment and force detection, opening the door to a variety of more complex configurations, including the recent measurement of the Casimir force between two metal-coated spheres [26].

Experimental advances to study more complex geometries have been guided by the theoretical potential of new geometries to give rise to repulsive forces or situations where analytical approximations begin to break down. One of the earliest geometries that appeared to give rise to a repulsive interaction is the hollow sphere [87]; however, closer inspection suggests that only attractive interactions should exist in this case [74, 82, 88]. More recently, a geometry has been proposed consisting of a needle-like metal object and a pinhole, which is predicted to lead to repulsion [89]. This result has led to the prediction of a class of systems that could yield repulsive forces [90]. 2D materials, such as those in the graphene family, have unique scaling laws for the Casimir energy compared to the traditional behavior of bulk metal plates. They also show promise for reducing the Casimir force when adhered onto a substrate in applications where sensitivity to the Casimir interaction may cause unwanted consequences [91–93]. While the development of new methods to detect the Casimir force is ongoing, significant advancements in technology are starting to allow experimentalists to explore these and other new geometries, which may lead to the measurement of a repulsive Casimir force between vacuum-separated materials.

4 Optical anisotropy and the Casimir torque

The expressions for the force between two surfaces derived by Casimir and Lifshitz assume that the interacting materials are optically isotropic; however, what would happen if these materials were optically *anisotropic* (see Figure 3)? In the early 1970s, Kats [94] and Parsegian and Weiss [95] independently answered this question. The expression derived by Parsegian and Weiss [95] is for the nonretarded interaction energy (which is generally valid for ≲10 nm and where the interaction is considered instantaneous) between two anisotropic dielectric plates, with in-plane optical axes that are parallel to the surface, immersed in a third medium. They found that the interaction energy depends on the angular orientation of the two plates, resulting in a torque that would cause their optical axes to align. In 1978, Barash [96] analyzed a similar problem using the Helmholtz free energy, which included retardation effects. In the nonretarded regime, both expressions are in agreement.

A number of proposals were made for experiments to detect this torque [97–102], and final experimental confirmation came in 2018 [32]. The difficulty in detecting this torque comes from the need to measure the rotation of one relatively large body in close proximity to another. For the case of two parallel, optically anisotropic plates, one could use a torsional rod to suspend one plate above the other; however, it is difficult to maintain parallelism of the plates at separations of less than 1 μm. To circumvent this obstacle, we made two modifications in our recent experiments [32]. First, we replaced the vacuum gap with an isotropic solid (Al_2O_3), which keeps the two plates separated at a fixed distance and parallel to each other without touching. The addition of this isotropic layer also increases the magnitude of the torque [103]. However, if all three layers are solids, frictional forces will prevent rotation.

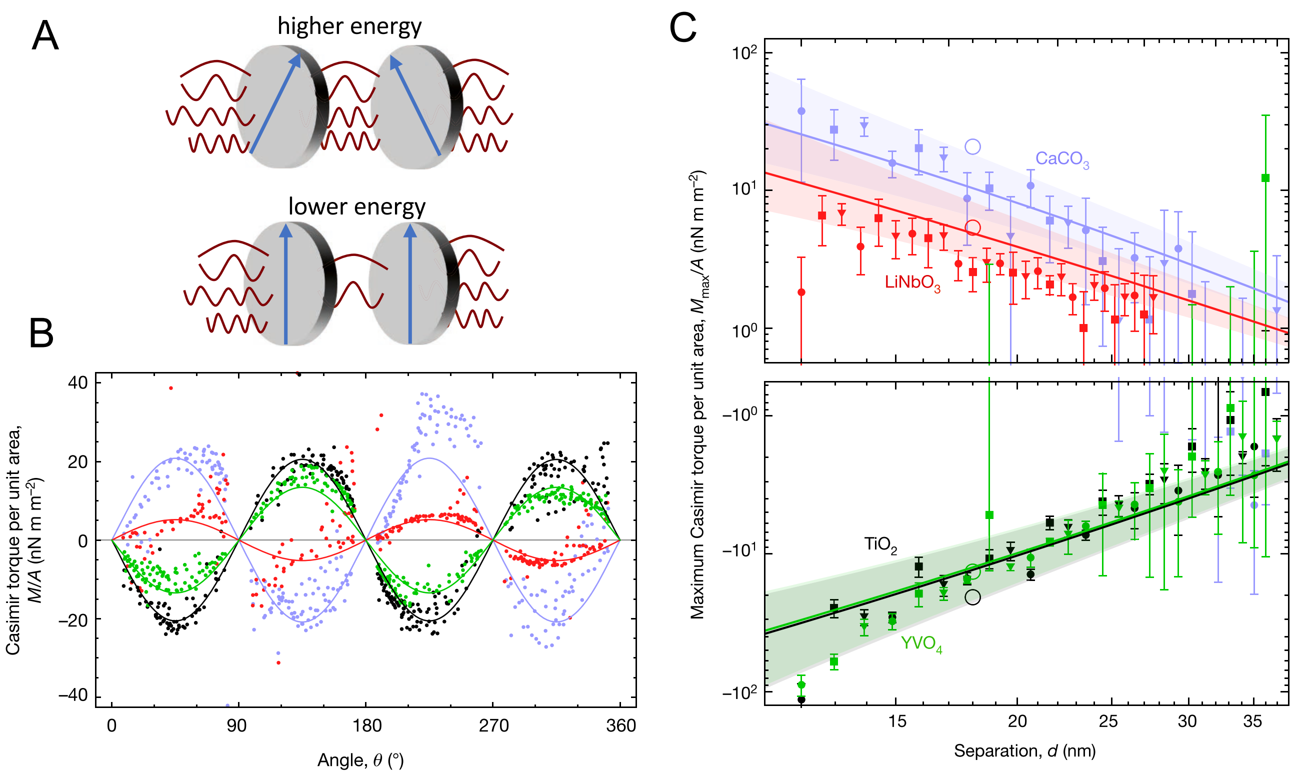

Figure 3: Casimir torque. (A) When materials with optical anisotropy are used instead of isotropic metal plates, the energy associated with the quantum fluctuations depends on their orientation and results in a torque. This Casimir torque is found to (B) have a sin(2θ) dependence and (C) decrease in magnitude with increasing separation between the plates. (B and C) Adapted with permission from the study by Somers et al. [32]. Copyright 2018, Nature Publishing Group.

Second, we replaced one of the solid anisotropic plates with a liquid crystal. The liquid crystal wets the Al_2O_3 layer and is free to rotate. This configuration allows for an all-optical measurement technique to detect the liquid crystal rotation caused by the Casimir torque (Figure 3). Further advances and new techniques to measure ultrasmall torques [104, 105] could potentially be used in future experiments to measure the Casimir torque between two solid objects or other rotational or quantum friction effects [106].

The Casimir torque unveils new engineering possibilities to create nanoscale torsional devices and gears based on quantum fluctuations. Because the torque depends on the optical properties of the interacting objects, there are further opportunities to tune this effect using materials with enhanced anisotropy [107, 108] or one-dimensional gratings [109]. Similarly, for some materials, different frequency components yield positive or negative contributions to the torque, suggesting that there could be a distance dependence in the sign of the torque for some materials or configurations [110], just as there can be a sign change in the force. All of these possibilities are now within experimental reach.

5 Nonequilibrium systems

The theories presented by both Casimir and Lifshitz assume all interacting objects are in thermal equilibrium with their surroundings; however, a variety of interesting phenomena can occur when this condition is relaxed (Figure 4A–C). An early approach by Henkel et al. [111] showed that for a particle–plate system where the plate is held at a finite temperature and the particle is held at 0 K, the particle will experience a temperature-dependent repulsive force attributed to the thermal fluctuations from the plate. Using a Rb Bose–Einstein condensate and a dielectric substrate, Obrecht et al. [112] were able to make the first measurement of these nonequilibrium forces and verify the previous theoretical predictions [113]. Interestingly, they found that by tuning the temperature of the substrate, the force due to the thermal fluctuations could either enhance the attraction or exert a repulsion.

For configurations involving two solid bodies at different temperatures, e.g., two spheres, a sphere and a plate [114], or two cylinders [115], not only has enhancement of the attraction or repulsion been predicted but also

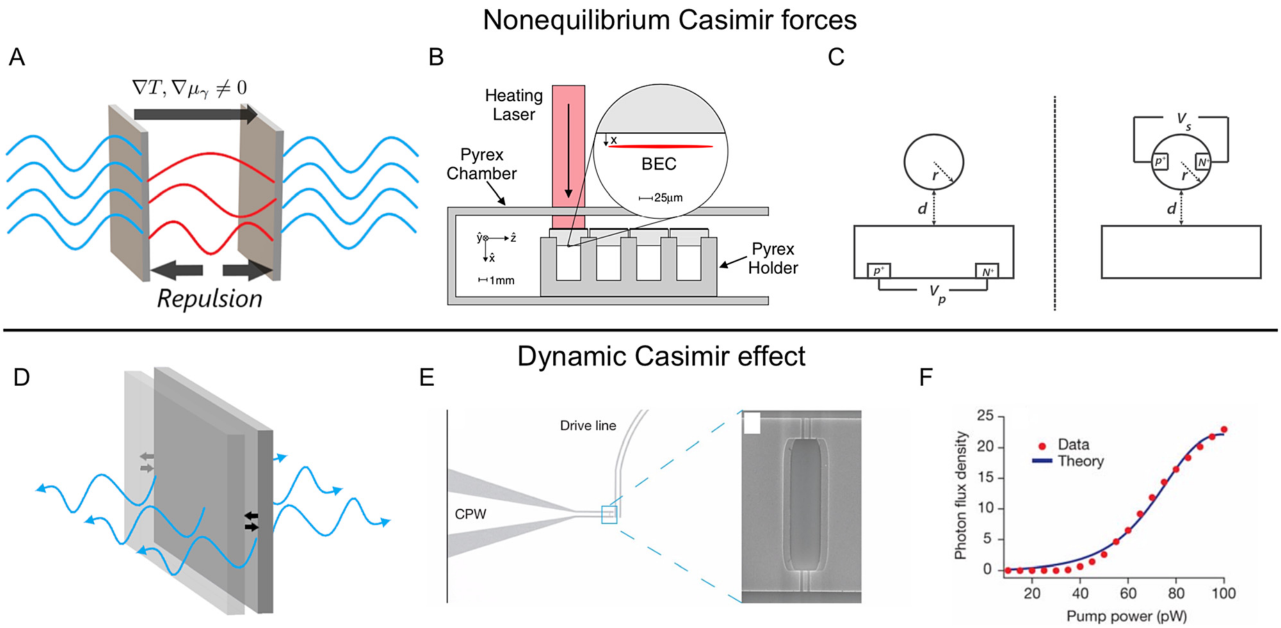

Figure 4: The Casimir effect beyond static equilibrium. (A) Schematic showing the system pushed out of equilibrium. (B) Experimental configuration showing the Casimir–Polder interaction between a Bose–Einstein condensate and a plate held at different temperatures. Reproduced with permission from the study by Obrecht et al. [112]. Copyright 2007, American Physical Society. (C) Configuration where photons are given a chemical potential based on the applied bias. Reproduced with permission from the study by Chen and Fan [116]. Copyright 2016, American Physical Society. (D) Schematic of the dynamic Casimir effect (DCE) showing the generation of photons resulting from an oscillating plate. (E) Schematic of a ~43-mm aluminum coplanar waveguide (CPW) terminated by a SQUID used to detect the DCE. The parametric inductance of the SQUID is tuned by applying a magnetic flux, which allows for a dynamically changing boundary condition. (F) Generated photon flux density due to the DCE at a frequency detuned from the central frequency (10.3 GHz) by 764 MHz with increasing pump power. (E and F) Reproduced with Permission from the study by Wilson et al. [117]. Copyright 2011, Nature Publishing Group.

a new oscillation in the force is expected as a result of scattering and interference of electromagnetic waves in the vicinity of the objects. This apparent oscillation between the attractive and repulsive regimes leads to multiple stable separations where the net force is zero. However, one major drawback of using a temperature difference to push the system out of equilibrium is that within the experimentally accessible (but still difficult) temperature difference of a few hundred Kelvin, the repulsive force is quite small. Further, the attractive equilibrium component typically dominates the interaction, especially at the nanoscale. A far greater temperature difference is needed for the repulsive, nonequilibrium component to dominate at the submicron regime.

One potential solution to the barriers faced using a temperature difference to induce a nonequilibrium state is to instead use an applied bias on one of the bodies to modify the chemical potential of the photons [116]. Calculations show, similar to a temperature difference, an applied bias to the plate modifies the chemical potential of the associated photons and can cause the sphere to experience a repulsion away from the plate. In the far field (beyond 1 μm), the nonequilibrium force shows no dependence on the sphere–plate separation and can simply be interpreted as radiation pressure. In the near field (down to separations as small as 50 nm), calculations predict a repulsion conveying the dominance of the nonequilibrium component well into the submicron regime. The magnitude of the nonequilibrium force calculated in this investigation is well within the measurement capability of current Casimir force techniques, suggesting the possibility of attaining a repulsive nonequilibrium Casimir force at nanoscale separations.

6 Generation of photons from vacuum

One interpretation of the Casimir effect is that it arises from the spatial mismatch of virtual photon modes inside and outside of a cavity. If the boundary of a cavity is oscillating sufficiently fast to render the process nonadiabatic, a temporal mismatch of the virtual photon modes inside the cavity occurs and real photons can be emitted from the cavity (Figure 4D). This process is referred to as the dynamic Casimir effect (DCE) [118].

The simplest system that can produce photons via the DCE is a single mirror in nonuniform motion [119]. The emitted photons in turn exert a radiation/damping force on the mirror, with a magnitude that is proportional to the time derivative of the acceleration of the mirror [120, 121]. If the mirror is oscillating at the frequency Ω with an amplitude of motion a, photons will be generated in pairs, and the emission spectrum is proportional to $\left(\frac{a}{c}\right)^2\omega(\Omega-\omega)$ based on a simplified 1D calculation and is symmetric about half the oscillation frequency [122]; see also the study by Neto and Machado [123] for the 3D case. The total number of generated photons within time T is found to vary quadratically with the maximum velocity of the oscillating mirror at the rate $\frac{\Omega}{3\pi}\left(\frac{v}{c}\right)^2$ [124].

Notwithstanding its theoretical interest, a single moving mirror can hardly produce experimentally detectable photons due to the limitation of the practically achievable mechanical oscillation speed (~10^{-8} c) and frequency (~3 GHz) [119]. However, it is predicted that by adding another parallel mirror to form a simple cavity, the photon generation rate can be enhanced by approximately the cavity Q-factor to be as high as 10^5 [125], which is attributed to a parametric enhancement effect [124]. Additionally, a number of studies have shown that when the intermode coupling condition is not satisfied, the number of generated photons grows exponentially over time as $N=\sinh^2\left(\Omega t\frac{v}{c}\right)$ under resonant oscillation $\Omega=2\omega_m$, where ω_m is the m^{th} resonance mode of the cavity [126]. Yet the enhanced photon generation rate (10^{-1}–10 s^{-1}) is still not adequate to be measured practically in this configuration.

To overcome the limitation imposed by mechanical oscillation, a time-varying boundary condition of a cavity can alternatively be induced with electromagnetic modulation, which gives rise to a modulation of the effective cavity length as a function of time. Various approaches along this direction have been proposed, most of which are focused on illuminating the cavity with short laser pulses to alter the dielectric function of the intracavity material (filled completely or partially with a semiconductor) [119, 127, 128]. In the microwave frequency range, semiconductors can behave like a reflective metal mirror under excitation by a laser pulse and become transparent after relaxing from excitation, which can result in oscillation lengths as large as a few millimeters [129–131]. A cavity inserted with nonlinear materials (e.g., $\chi^{(2)}$ or $\chi^{(3)}$ materials) with an effective refractive index that changes with applied light field has been proposed as another promising configuration to achieve measurable DCE photons [132, 133]. These aforementioned modulation methods are theoretically predicted to yield a much larger photon generation rate (10^5–10^{10} s^{-1}), enabling experimental detection. In recent years, other novel cavity modulation methods have drawn increasing attention. In particular, a superconducting quantum interference device (SQUID) modulated by a magnetic field flux can mimic a moving mirror for generating DCE photons, which has experimentally been realized in the microwave regime [117, 134] (see Figure 4E, F), demonstrating great promises of extracting the vacuum photons consistently and reliably. Very recently, an analog DCE has been realized in the near-infrared regime using a dispersion-oscillating photonic crystal fiber [135].

7 Vacuum fluctuation chemistry

Apart from the Casimir effect, vacuum field fluctuations have been attributed to a number of other nonclassical phenomena, among which their significant influence on chemistry and materials science has been gaining attention. As an example, the strong coupling between vacuum fields in an optical cavity and electronic transitions in a molecule can give rise to two hybrid polaritonic states (P+ and P−), separated by the Rabi splitting energy (Figure 5A). In organic molecules, the splitting can be as large as hundreds of meV due to the large transition dipole moments and can hence modify the energy landscape of the reactant molecules. The strong coupling with vacuum fields is found to dramatically influence the kinetics of the photoisomeric conversion from spiropyran into merocyanine (MC) [136]. When the cavity resonance is tuned with the MC absorption at 560 nm, the photoisomeric reaction rate is drastically reduced but the final MC yield is increased by 10% (Figure 5B, C). The modified energy landscape of a molecule can also be expected to alter the electron affinity and work function and may find applications in optoelectronic device design such as light-emitting diodes (LEDs) and photovoltaics.

Molecular vibrational modes are also able to strongly couple with vacuum field fluctuations in a microcavity at room temperature, resulting in Rabi splitting and a change of the Morse potential and the resultant chemical reactivity [138–140]. Vibrational strong coupling (VSC) has drawn increasing interest as the role of the vibrational modes is decisive in determining the kinetics of isomeric and other chemical processes, and the potential impact on bond-selective chemistry is significant. For example, the ground-state deprotection reaction of an alkynylsilane,

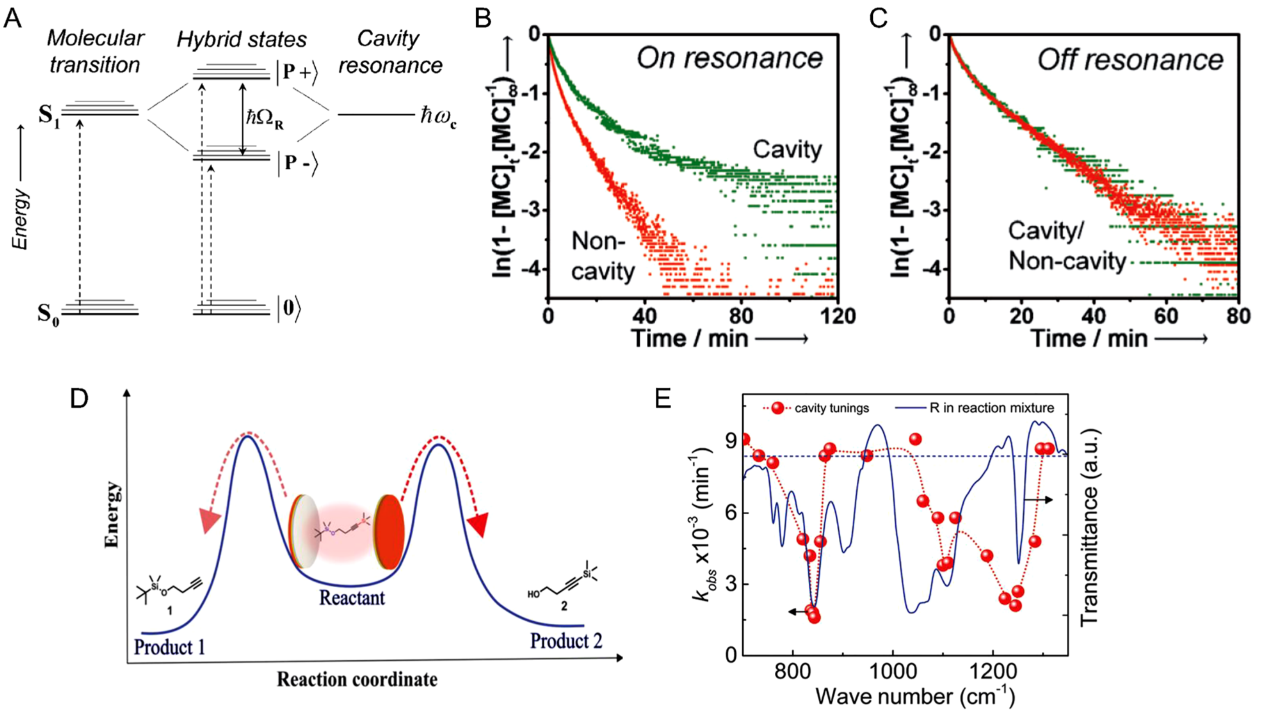

Figure 5: Vacuum fluctuations affecting chemistry. (A) Schematic of the hybridization of energy states when the molecular electron transitions are strongly coupled with vacuum fluctuations in a cavity. The kinetics of the photoisomeric conversion from spiropyran (SPI) into merocyanine (MC) measured for bare molecules (red) and molecules coupled with cavity vacuum fluctuation (green) in both (B) on-resonance (when cavity is tuned on resonance with the MC absorption wavelength at 560 nm) and (C) off-resonance cases. (D) Schematic of two silyl cleavage pathways for a silane derivative with tetrabutylammonium fluoride in a 1:1 mixture of methanol and tetrahydrofuran. (E) The total reaction rate as a function of cavity tuning (red dots) is modified from the rate outside of the cavity (blue dashed line) when the cavity is tuned in resonance with the transmission dips (solid blue line) that correspond to the vibrational modes from the infrared absorption spectrum. (A, B, and C) Reproduced with permission from the study by Hutchison et al. [136]. Copyright 2012, Wiley-VCH. (D and E) Reproduced with permission from the study by Thomas et al. [137]. Copyright 2019, The American Association for the Advancement of Science.

1-phenyl-2-trimethylsilylacetylene (PTA) placed inside a microfluid microcavity is found to be slowed by a factor of 5 when the S–C vibrational stretching mode of the reactant is strongly coupled with the cavity vacuum fields [141]. Further, site selectivity can be modified in reactions with multiple final products so that one product can be favored over the others. As an example, two distinct sites for prospective silyl bond cleavage in a silane derivative (see Figure 5D) can be selectively excited by tuning the cavity (two parallel Au-coated ZnSe mirrors) into resonances corresponding to the two distinct vibrational modes (Si–C stretching bond at 842 cm^{-1} and Si–O stretching bond at 110 cm^{-1}) [137]. When the Si–C stretching bond vibration is strongly coupled with the cavity vacuum field, the total reaction rate is reduced by a factor of 3.5, whereas the overall rate is slowed by a factor of 2.5 when the Si–O stretching vibration is strongly coupled (Figure 5E). More interesting, the ratio of the final products indicates that in either case the VSC modifies the reaction landscape to favor the Si–O cleavage over the Si–C cleavage when off-resonance or outside the cavity, which reverses the selectivity. Vacuum fluctuations can help to control the chemical landscape and aid in a better understanding of the reaction mechanisms for a variety of chemical reactions, including effects in biochemistry and enzyme catalysis [142].

8 Applications to nanotechnology

Because the Casimir force has a strong power law dependence with separation, it becomes the dominant interaction between objects on the nanoscale. For example, the pressure resulting from the Casimir force between two parallel plates separated by 10 nm is approximately 1 atm (or 10^5 N/m^2). Thus, the Casimir force provides both an impediment and an opportunity for nanoscale devices [48, 143].

Advances in silicon processing and integrated circuit technology have allowed for the development of advanced MEMS. These devices can be found in a variety of

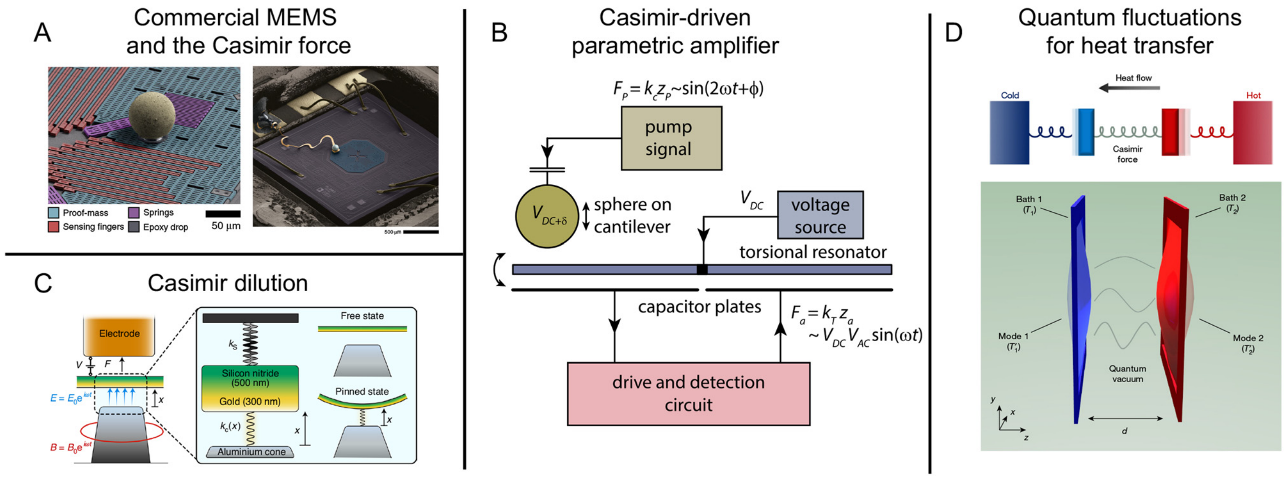

Figure 6: Application of the Casimir force to nanotechnology. (A) Use of a commercial MEMS sensor to measure the Casimir force. Reproduced with permission from the study by Stange et al. [150]. Copyright 2019, Nature Publishing Group. (B) Casimir parametric amplifier. Reproduced with permission from the study by Imboden et al. [151]. Copyright 2014, AIP Publishing. (C) Casimir force incorporated into an optomechanical cavity giving rise to dissipation dilution. Reproduced with permission from the study by Pate et al. [152]. Copyright 2020, Nature Publishing Group. (D) Heat transfer driven by quantum fluctuations. Reproduced with permission from the study by Fong et al. [153]. Copyright 2019, Nature Publishing Group.

technologies ranging from airbag sensors to cell phones. With a push toward further miniaturization (and the transition from MEMS to NEMS), movable parts in these systems begin to experience the Casimir force, which can lead to stiction and adhesion if not properly designed around [48, 144]. Alternatively, the Casimir effect can be advantageous in these devices due to its sensitivity to small changes in distance – examples include actuators [55], devices exploiting bistability and hysteresis [145–147], chaotic behavior [148], virtually frictionless bearings [149], etcetera.

Recently, it has been shown that an off-the-shelf MEMS sensor can be used to measure the Casimir force [150], which shows how commercial technologies and the Casimir effect are beginning to overlap (Figure 6A). Another example is a Casimir-driven parametric amplifier [151], see Figure 6B. This concept involves using the Casimir force to modulate a MEMS parametric amplifier to enable a very sensitive voltage measurement technique, which could be integrated into other sensing devices.

The Casimir force has also found its way into cavity optomechanics and nanoscale heat transfer. One example is the use of the Casimir force to engineer dilution in a macroscopic system involving a SiN membrane and a photonic cavity [152]. In this case, the Casimir force causes an additional strain on the acoustic membrane, allowing the mechanical Q factor to be increased (Figure 6C). In another example, it has been found that heat can be transferred through quantum fluctuations, rather than through the conventional routes of conduction, convection, and radiation [153], see Figure 6D. This effect is the result of induced phonon coupling across a vacuum gap due to the electromagnetic fluctuations. This new method of heat transfer could play a significant role in thermal management of future nanoscale technologies.

9 Conclusions

Quantum fluctuations are responsible for a number of phenomena, which we can begin to engineer through material properties, geometry, temperature, etcetera. As we continue to pursue these research directions, the lines between optics, physics, and chemistry blur as these fields coalesce and applications emerge in nanoscale devices, catalytic behavior, and novel chemistry. Herein, we have outlined several of the recent developments in Casimir physics and related fields and point to future directions and developments that we believe will have a significant impact on science and technology in the coming decades through the engineering of quantum vacuum fluctuations.

Acknowledgments: The authors would like to thank Joseph Garrett and Marina Leite for helpful discussions and suggestions pertaining to the manuscript.
Author contribution: All the authors have accepted responsibility for the entire content of this submitted manuscript and approved submission.

Research funding: The authors acknowledge funding from a DARPA YFA Grant No. D18AP00060, a DARPA QUEST grant No. HR00112090084, and the National Science Foundation under grant number PHY-1806768.
Conflict of interest statement: The authors declare no conflicts of interest regarding this article.

References

[1] P. W. Milonni, *The Quantum Vacuum: An Introduction to Quantum Electrodynamics*, 1st ed., Boston, US, Academic Press, 1993.
[2] J. N. Israelachvili, *Intermolecular and Surface Forces*, 3rd ed., Academic Press, 2011.
[3] V. A. Parsegian, *Van der Waals Forces: A Handbook for Biologists, Chemists, Engineers, and Physicists*, New York, Cambridge University Press, 2005.
[4] K. Autumn, Y. A. Liang, S. T. Hsieh, et al., "Adhesive force of a single gecko foot-hair," *Nature*, vol. 405, pp. 681–685, 2000.
[5] H. B. G. Casimir, "On the attraction between two perfectly conducting plates," *Proc. K. Ned. Akad.* vol. 360, pp. 793–795, 1948. https://www.dwc.knaw.nl/DL/publications/PU00018547.pdf.
[6] I. I. Abrikosova, B. V. Deriagin, "Direct measurement of molecular attraction of solid bodies. II. Method for measuring the gap. Results of experiments," *Sov. Phys. JETP* vol. 4, pp. 2–10, 1957. http://www.jetp.ac.ru/cgi-bin/e/index/e/4/1/p2?a=list.
[7] M. J. Sparnaay, "Measurements of attractive forces between flat plates," *Physica*, vol. 24, pp. 751–764, 1958.
[8] P. H. G. M. van Blokland, and J. T. G. Overbeek, "van der Waals forces between objects covered with a chromium layer," *J. Chem. Soc. Faraday Trans.*, vol. 74, pp. 2637–2651, 1978.
[9] S. K. Lamoreaux, "Demonstration of the Casimir force in the 0.6 to 6 μm range," *Phys. Rev. Lett.*, vol. 78, pp. 5–8, 1997.
[10] U. Mohideen and A. Roy, "Precision measurement of the Casimir force from 0.1 to 0.9 μm," *Phys. Rev. Lett.*, vol. 81, pp. 4549–4552, 1998.
[11] A. A. Maradudin and P. Mazur, "Effects of surface roughness on the van der Waals force between macroscopic bodies," *Phys. Rev. B*, vol. 22, pp. 1677–1686, 1980.
[12] V. B. Bezerra, G. L. Klimchitskaya, and C. Romero, "Casimir force between a flat plate and a spherical lens: application to the results of a new experiment," *Mod. Phys. Lett. A*, vol. 12, pp. 2613–2622, 1997.
[13] P. A. Maia Neto, A. Lambrecht, and S. Reynaud, "Roughness correction to the Casimir force: beyond the proximity force approximation," *Europhys. Lett.*, vol. 100, p. 29902, 2012.
[14] P. J. van Zwol, G. Palasantzas, and J. T. M. De Hosson, "Influence of random roughness on the Casimir force at small separations," *Phys. Rev. B*, vol. 77, p. 075412, 2008.
[15] J. L. Garrett, J. Kim, and J. N. Munday, "Measuring the effect of electrostatic patch potentials in Casimir force experiments," *Phys. Rev. Res.*, vol. 2, p. 023355, 2020.
[16] W. J. Kim, M. Brown-Hayes, D. A. R. Dalvit, J. H. Brownell, and R. Onofrio, "Anomalies in electrostatic calibrations for the measurement of the Casimir force in a sphere-plane geometry," *Phys. Rev. A*, vol. 78, p. 020101(R), 2008.
[17] R. O. Behunin, F. Intravaia, D. A. R. Dalvit, P. A. M. Neto, and S. Reynaud, "Modeling electrostatic patch effects in Casimir force measurements," *Phys. Rev. A*, vol. 85, p. 012504, 2012.
[18] D. Garcia-Sanchez, K. Y. Fong, H. Bhaskaran, S. Lamoreaux, and H. X. Tang, "Casimir force and in situ surface potential measurements on nanomembranes," *Phys. Rev. Lett.*, vol. 109, p. 027202, 2012.
[19] R. O. Behunin, D. A. R. Dalvit, R. S. Decca, et al., "Kelvin probe force microscopy of metallic surfaces used in Casimir force measurements," *Phys. Rev. A*, vol. 90, p. 062115, 2014.
[20] R. O. Behunin, D. A. R. Dalvit, R. S. Decca, and C. C. Speake, "Limits on the accuracy of force sensing at short separations due to patch potentials," *Phys. Rev. D*, vol. 89, p. 051301, 2014.
[21] J. L. Garrett, D. Somers, and J. N. Munday, "The effect of patch potentials in Casimir force measurements determined by heterodyne Kelvin probe force microscopy," *J. Phys. Condens. Matter*, vol. 27, p. 214012, 2015.
[22] A. O. Sushkov, W. J. Kim, D. A. R. Dalvit, and S. K. Lamoreaux, "Observation of the thermal Casimir force," *Nat. Phys.*, vol. 7, pp. 230–233, 2011.
[23] G. Bimonte, D. López, and R. S. Decca, "Isoelectronic determination of the thermal Casimir force," *Phys. Rev. B*, vol. 93, p. 184434, 2016.
[24] G. Bressi, G. Carugno, R. Onofrio, and G. Ruoso, "Measurement of the Casimir force between parallel metallic surfaces," *Phys. Rev. Lett.*, vol. 88, p. 041804, 2002.
[25] M. Brown-Hayes, D. A. R. Dalvit, F. D. Mazzitelli, W. J. Kim, and R. Onofrio, "Towards a precision measurement of the Casimir force in a cylinder-plane geometry," *Phys. Rev. A*, vol. 72, pp. 1–11, 2005.
[26] J. L. Garrett, D. A. T. Somers, and J. N. Munday, "Measurement of the Casimir force between two spheres," *Phys. Rev. Lett.*, vol. 120, p. 040401, 2018.
[27] W. J. Kim, A. O. Sushkov, D. A. R. Dalvit, and S. K. Lamoreaux, "Measurement of the short-range attractive force between Ge plates using a torsion balance," *Phys. Rev. Lett.*, vol. 103, p. 060401, 2009.
[28] F. Intravaia, S. Koev, I. W. Jung, et al., "Strong Casimir force reduction through metallic surface nanostructuring," *Nat. Commun.*, vol. 4, p. 2515, 2013.
[29] F. Chen, U. Mohideen, G. L. Klimchitskaya, and V. M. Mostepanenko, "Demonstration of the lateral Casimir force," *Phys. Rev. Lett.*, vol. 88, p. 101801, 2002.
[30] L. Tang, M. Wang, C. Y. Ng, et al., "Measurement of non-monotonic Casimir forces between silicon nanostructures," *Nat. Photonics*, vol. 11, pp. 97–101, 2017.
[31] D. E. Krause, R. S. Decca, D. López, and E. Fischbach, "Experimental investigation of the Casimir force beyond the proximity-force approximation," *Phys. Rev. Lett.*, vol. 98, p. 050403, 2007.
[32] D. A. T. Somers, J. L. Garrett, K. J. Palm, and J. N. Munday, "Measurement of the Casimir torque," *Nature*, vol. 564, pp. 386–389, 2018.
[33] D. Iannuzzi, M. Lisanti, and F. Capasso, "Effect of hydrogen-switchable mirrors on the Casimir force," *Proc. Natl. Acad. Sci. U. S. A.*, vol. 101, p. 4019, 2004.
[34] A. Le Cunuder, A. Petrosyan, G. Palasantzas, V. Svetovoy, and S. Ciliberto, "Measurement of the Casimir force in a gas and in a liquid," *Phys. Rev. B*, vol. 98, p. 201408, 2018.

[35] R. S. Decca, D. Lopez, E. Fischbach, and D. E. Krause, "Measurement of the Casimir force between dissimilar metals," *Phys. Rev. Lett.*, vol. 91, p. 050402, 2003.
[36] S. de Man, K. Heeck, R. J. Wijngaarden, and D. Iannuzzi, "Halving the Casimir force with conductive oxides," *Phys. Rev. Lett.*, vol. 103, p. 040402, 2009.
[37] A. A. Banishev, C. C. Chang, G. L. Klimchitskaya, V. M. Mostepanenko, and U. Mohideen, "Measurement of the gradient of the Casimir force between a nonmagnetic gold sphere and a magnetic nickel plate," *Phys. Rev. B*, vol. 85, p. 195422, 2012.
[38] A. A. Banishev, H. Wen, J. Xu, et al., "Measuring the Casimir force gradient from graphene on a SiO_2 substrate," *Phys. Rev. B*, vol. 87, p. 205433, 2013.
[39] A. A. Banishev, G. L. Klimchitskaya, V. M. Mostepanenko, and U. Mohideen, "Demonstration of the Casimir force between ferromagnetic surfaces of a Ni-coated sphere and a Ni-coated plate," *Phys. Rev. Lett.*, vol. 110, p. 137401, 2013.
[40] P. W. Milonni and M.-L. Shih, "Casimir forces," *Contemp. Phys.*, vol. 33, pp. 313–322, 1992.
[41] L. Spruch, "Long-range (Casimir) interactions," *Science*, vol. 272, p. 1452, 1996.
[42] V. Mostepanenko and N. N. Trunov, *The Casimir Effect and Its Applications*, 1st ed., Oxford, U.K., Clarendon Press, 1997.
[43] K. A. Milton, *The Casimir Effect: Physical Manifestations of Zero-point Energy*, River Edge, USA, World Scientific, 2001.
[44] M. Bordag, U. Mohideen, and V. M. Mostepanenko, "New developments in the Casimir effect," *Phys. Rep.*, vol. 353, pp. 1–205, 2001.
[45] P. A. Martin, P. R. Buenzli, "The Casimir effect," *Acta Phys. Pol. B* vol. 37, pp. 2503–2559, 2006. https://www.actaphys.uj.edu.pl/R/37/9/2503.
[46] A. Lambrecht, "The Casimir effect: a force from nothing," *Phys. World*, vol. 15, pp. 29–32, 2002.
[47] S. K. Lamoreaux, "Resource letter CF-1: Casimir force," *Am. J. Phys.*, vol. 67, pp. 850–861, 1999.
[48] F. Capasso, J. N. Munday, D. Iannuzzi, and H. B. Chan, "Casimir forces and quantum electrodynamical torques: physics and nanomechanics," *IEEE J. Sel. Top. Quant.*, vol. 13, pp. 400–414, 2007.
[49] A. W. Rodriguez, F. Capasso, and S. G. Johnson, "The Casimir effect in microstructured geometries," *Nat. Photonics*, vol. 5, pp. 211–221, 2011.
[50] L. M. Woods, D. A. R. Dalvit, A. Tkatchenko, P. Rodriguez-Lopez, A. W. Rodriguez, and R. Podgornik, "Materials perspective on Casimir and van der Waals interactions," *Rev. Mod. Phys.*, vol. 88, p. 045003, 2016.
[51] D. Iannuzzi, M. Lisanti, J. N. Munday, and F. Capasso, "The design of long range quantum electrodynamical forces and torques between macroscopic bodies," *Solid State Commun.*, vol. 135, pp. 618–626, 2005.
[52] A. Roy, C.-Y. Lin, and U. Mohideen, "Improved precision measurement of the Casimir force," *Phys. Rev. D*, vol. 60, p. 111101, 1999.
[53] B. W. Harris, H. F. Chen, and U. Mohideen, "Precision measurement of the Casimir force using gold surfaces," *Phys. Rev. A*, vol. 62, p. 052109–052101, 2000.
[54] R. S. Decca, D. López, E. Fischbach, G. L. Klimchitskaya, D. E. Krause, and V. M. Mostepanenko, "Precise comparison of theory and new experiment for the Casimir force leads to stronger constraints on thermal quantum effects and long-range interactions," *Ann. Phys.*, vol. 318, pp. 37–80, 2005.
[55] H. B. Chan, V. A. Aksyuk, R. N. Kleiman, D. J. Bishop, and F. Capasso, "Quantum mechanical actuation of microelectromechanical systems by the Casimir force," *Science*, vol. 291, pp. 1941–1944, 2001.
[56] S. de Man, K. Heeck, and D. Iannuzzi, "Halving the Casimir force with conductive oxides: experimental details," *Phys. Rev. A*, vol. 82, p. 062512, 2010.
[57] J. L. Garrett, D. A. T. Somers, K. Sendgikoski, and J. N. Munday, "Sensitivity and accuracy of Casimir force measurements in air," *Phys. Rev. A*, vol. 100, p. 022508, 2019.
[58] J. N. Munday, F. Capasso, V. A. Parsegian, and S. M. Bezrukov, "Measurements of the Casimir-Lifshitz force in fluids: the effect of electrostatic forces and Debye screening," *Phys. Rev. A*, vol. 78, p. 032109, 2008.
[59] J. N. Munday, F. Capasso, and V. A. Parsegian, "Measured long-range repulsive Casimir-Lifshitz forces," *Nature*, vol. 457, pp. 170–173, 2009.
[60] G. Torricelli, P. J. van Zwol, O. Shpak, et al., "Switching Casimir forces with phase-change materials," *Phys. Rev. A*, vol. 82, p. 010101, 2010.
[61] H. B. Chan, Y. Bao, J. Zou, et al., "Measurement of the Casimir force between a gold sphere and a silicon surface with nanoscale trench arrays," *Phys. Rev. Lett.*, vol. 101, pp. 1–4, 2008.
[62] E. M. Lifshitz, "The theory of molecular attractive forces between solids," *Sov. Phys. JETP-USSR* vol. 2, pp. 73–83, 1956. http://www.jetp.ac.ru/cgi-bin/dn/e_002_01_0073.pdf.
[63] G. Bimonte, E. Calloni, G. Esposito, L. Milano, and L. Rosa, "Towards measuring variations of Casimir energy by a superconducting cavity," *Phys. Rev. Lett.*, vol. 94, p. 180402, 2005.
[64] E. G. Galkina, B. A. Ivanov, S. Savel'ev, V. A. Yampol'skii, and F. Nori, "Drastic change of the Casimir force at the metal-insulator transition," *Phys. Rev. B*, vol. 80, p. 125119, 2009.
[65] L. H. Ford, "Spectrum of the Casimir effect," *Phys. Rev. D*, vol. 38, pp. 528–532, 1988.
[66] A. Rodriguez, M. Ibanescu, D. Iannuzzi, J. D. Joannopoulos, and S. G. Johnson, "Virtual photons in imaginary time: computing exact Casimir forces via standard numerical electromagnetism techniques," *Phys. Rev. A*, vol. 76, p. 032106, 2007.
[67] U. Leonhardt and T. G. Philbin, "Quantum levitation by left-handed metamaterials," *New J. Phys.*, vol. 9, pp. 254, 2007.
[68] R. Zhao, J. Zhou, T. Koschny, E. N. Economou, and C. M. Soukoulis, "Repulsive Casimir force in chiral metamaterials," *Phys. Rev. Lett.*, vol. 103, p. 103602, 2009.
[69] A. P. McCauley, R. Zhao, M. T. H. Reid, et al., "Microstructure effects for Casimir forces in chiral metamaterials," *Phys. Rev. B*, vol. 82, 2010, https://doi.org/10.1103/physrevb.82.165108.
[70] F. S. S. Rosa, "On the possibility of Casimir repulsion using Metamaterials," *J. Phys. Conf. Ser.*, vol. 161, p. 012039, 2009.
[71] F. S. S. Rosa, D. A. R. Dalvit, and P. W. Milonni, "Casimir-lifshitz theory and metamaterials," *Phys. Rev. Lett.*, vol. 100, p. 183602, 2008.
[72] A. W. Rodriguez, A. P. McCauley, J. D. Joannopoulos, and S. G. Johnson, "Casimir forces in the time domain: theory," *Phys. Rev. A*, vol. 80, p. 012115, 2009.
[73] L. D. Landau, E. M. Lifshitz, and L. P. Pitaevskii, *Electrodynamics of Continuous Media*, 2nd ed., Oxford, Pergamon Press, 1960.

[74] O. Kenneth and I. Klich, “Opposites attract: a theorem about the Casimir force,” *Phys. Rev. Lett.*, vol. 97, p. 160401, 2006.
[75] I. E. Dzyaloshinskii, E. M. Lifshitz, and L. P. Pitaevskii, “The general theory of van der Waals forces,” *Adv. Phys.*, vol. 10, pp. 165–209, 1961.
[76] A. A. Feiler, L. Bergström, and M. W. Rutland, “Superlubricity using repulsive van der Waals forces,” *Langmuir*, vol. 24, pp. 2274–2276, 2008.
[77] A. W. Rodriguez, J. N. Munday, J. D. Joannopoulos, F. Capasso, D. A. R. Dalvit, and S. G. Johnson, “Stable suspension and dispersion-induced transitions from repulsive Casimir forces between fluid-separated eccentric cylinders,” *Phys. Rev. Lett.*, vol. 101, p. 190404, 2008.
[78] R. K. Zhao, L. Li, S. Yang, et al., “Stable Casimir equilibria and quantum trapping,” *Science*, vol. 364, pp. 984–987, 2019.
[79] B. Spreng, P. A. Maia Neto, and G.-L. Ingold, “Plane-wave approach to the exact van der Waals interaction between colloid particles,” *J. Chem. Phys.*, vol. 153, p. 024115, 2020.
[80] D. A. T. Somers and J. N. Munday, “Conditions for repulsive Casimir forces between identical birefringent materials,” *Phys. Rev. A*, vol. 95, p. 022509, 2017.
[81] T. H. Boyer, “Van der Waals forces and zero-point energy for dielectric and permeable materials,” *Phys. Rev. A*, vol. 9, pp. 2078–2084, 1974.
[82] O. Kenneth, I. Klich, A. Mann, and M. Revzen, “Repulsive Casimir forces,” *Phys. Rev. Lett.*, vol. 89, p. 033001, 2002.
[83] D. Iannuzzi and F. Capasso, “Comment on “Repulsive Casimir forces”,” *Phys. Rev. Lett.*, vol. 91, p. 029101, 2003.
[84] O. Kenneth, I. Klich, A. Mann, and M. Revzen, “Kenneth et al. reply,” *Phys. Rev. Lett.*, vol. 91, p. 029102, 2003.
[85] V. Yannopapas and N. V. Vitanov, “First-principles study of Casimir repulsion in metamaterials,” *Phys. Rev. Lett.*, vol. 103, p. 120401, 2009.
[86] I. Liberal and N. Engheta, “Zero-index structures as an alternative platform for quantum optics,” *Proc. Natl. Acad. Sci. U. S. A.*, vol. 114, p. 822, 2017.
[87] T. H. Boyer, “Quantum electromagnetic zero-point energy of a conducting spherical shell and the Casimir model for a charged particle,” *Phys. Rev.*, vol. 174, pp. 1764–1776, 1968.
[88] R. L. Jaffe, “Unnatural acts: unphysical consequences of imposing boundary conditions on quantum fields,” *AIP Conf. Proc* vol. 687, pp. 3–12, 2003.
[89] M. Levin, A. P. McCauley, A. W. Rodriguez, M. T. H. Reid, and S. G. Johnson, “Casimir repulsion between metallic objects in vacuum,” *Phys. Rev. Lett.*, vol. 105, 2010, https://doi.org/10.1103/physrevlett.105.090403.
[90] P. S. Venkataram, S. Molesky, P. Chao, and A. W. Rodriguez, “Fundamental limits to attractive and repulsive Casimir-Polder forces,” *Phys. Rev. A*, vol. 101, p. 052115, 2020.
[91] S. Tsoi, P. Dev, A. L. Friedman, et al., “van der Waals screening by single-layer graphene and molybdenum disulfide,” *ACS Nano*, vol. 8, pp. 12410–12417, 2014.
[92] P. Rodriguez-Lopez, W. J. M. Kort-Kamp, D. A. R. Dalvit, and L. M. Woods, “Casimir force phase transitions in the graphene family,” *Nat. Commun.*, vol. 8, pp. 1–9, 2017.
[93] G. Bimonte, G. L. Klimchitskaya, and V. M. Mostepanenko, “How to observe the giant thermal effect in the Casimir force for graphene systems,” *Phys. Rev. A*, vol. 96, p. 012517, 2017.
[94] E. I. Kats, “Van der Waals forces in non-isotropic systems,” *Sov. Phys. JETP*, vol. 33, p. 634, 1971.
[95] V. A. Parsegian and G. H. Weiss, “Dielectric anisotropy and the van der Waals interaction between bulk media,” *J. Adhes.*, vol. 3, pp. 259–267, 1972.
[96] Y. S. Barash, “Moment of van der Waals forces between anisotropic bodies,” *Radiophys. Quantum Electron.*, vol. 21, pp. 1138–1143, 1978.
[97] J. N. Munday, D. Iannuzzi, Y. Barash, and F. Capasso, “Torque on birefringent plates induced by quantum fluctuations,” *Phys. Rev. A*, vol. 71, p. 042102, 2005.
[98] J. N. Munday, D. Iannuzzi, and F. Capasso, “Quantum electrodynamical torques in the presence of Brownian motion,” *New J. Phys.*, vol. 8, pp. 244, 2006.
[99] R. B. Rodrigues, P. A. M. Neto, A. Lambrecht, and S. Reynaud, “Vacuum-induced torque between corrugated metallic plates,” *Europhys. Lett.*, vol. 76, pp. 822–828, 2006.
[100] X. Chen and J. C. H. Spence, “On the measurement of the Casimir torque,” *Phys. Status Solidi B*, vol. 248, pp. 2064–2071, 2011.
[101] R. Guérout, C. Genet, A. Lambrecht, and S. Reynaud, “Casimir torque between nanostructured plates,” *Europhys. Lett.*, vol. 111, p. 44001, 2015.
[102] D. A. T. Somers and J. N. Munday, “Rotation of a liquid crystal by the Casimir torque,” *Phys. Rev. A*, vol. 91, p. 032520, 2015.
[103] D. A. T. Somers and J. N. Munday, “Casimir-lifshitz torque enhancement by retardation and intervening dielectrics,” *Phys. Rev. Lett.*, vol. 119, p. 183001, 2017.
[104] J. Ahn, Z. Xu, J. Bang, et al., “Optically levitated nanodumbbell torsion balance and GHz nanomechanical rotor,” *Phys. Rev. Lett.*, vol. 121, p. 033603, 2018.
[105] J. Ahn, Z. Xu, J. Bang, P. Ju, X. Gao, and T. Li, “Ultrasensitive torque detection with an optically levitated nanorotor,” *Nat. Nanotechnol.*, vol. 15, pp. 89–93, 2020.
[106] R. Zhao, A. Manjavacas, F. J. García de Abajo, and J. B. Pendry, “Rotational quantum friction,” *Phys. Rev. Lett.*, vol. 109, p. 123604, 2012.
[107] S. Niu, G. Joe, H. Zhao, et al., “Giant optical anisotropy in a quasi-one-dimensional crystal,” *Nat. Photonics*, vol. 12, pp. 392–396, 2018.
[108] A. Segura, L. Artús, R. Cuscó, T. Taniguchi, G. Cassabois, and B. Gil, “Natural optical anisotropy of h-BN: highest giant birefringence in a bulk crystal through the mid-infrared to ultraviolet range,” *Phys. Rev. Mater.*, vol. 2, p. 024001, 2018.
[109] M. Antezza, H. B. Chan, B. Guizal, V. N. Marachevsky, R. Messina, and M. Wang, “Giant Casimir torque between rotated gratings and the $\theta = 0$ anomaly,” *Phys. Rev. Lett.*, vol. 124, p. 013903, 2020.
[110] P. Thiyam, P. Parashar, K. V. Shajesh, et al., “Distance-dependent sign reversal in the Casimir-lifshitz torque,” *Phys. Rev. Lett.*, vol. 120, p. 131601, 2018.
[111] C. Henkel, K. Joulain, J.-P. Mulet, and J.-J. Greffet, “Radiation forces on small particles in thermal near fields,” *J. Opt. A*, vol. 4, pp. S109–S114, 2002.
[112] J. M. Obrecht, R. J. Wild, M. Antezza, L. P. Pitaevskii, S. Stringari, and E. A. Cornell, “Measurement of the temperature dependence of the Casimir-polder force,” *Phys. Rev. Lett.*, vol. 98, p. 063201, 2007.

[113] M. Antezza, L. P. Pitaevskii, and S. Stringari, "New asymptotic behavior of the surface-atom force out of thermal equilibrium," *Phys. Rev. Lett.*, vol. 95, p. 113202, 2005.

[114] M. Krüger, T. Emig, G. Bimonte, and M. Kardar, "Non-equilibrium Casimir forces: spheres and sphere-plate," *Europhys. Lett.*, vol. 95, p. 21002, 2011.

[115] V. A. Golyk, M. Krüger, M. T. H. Reid, and M. Kardar, "Casimir forces between cylinders at different temperatures," *Phys. Rev. D*, vol. 85, p. 065011, 2012.

[116] K. Chen and S. Fan, "Nonequilibrium Casimir force with a nonzero chemical potential for photons," *Phys. Rev. Lett.*, vol. 117, p. 267401, 2016.

[117] C. M. Wilson, G. Johansson, A. Pourkabirian, et al., "Observation of the dynamical Casimir effect in a superconducting circuit," *Nature*, vol. 479, pp. 376–379, 2011.

[118] G. T. Moore, "Quantum theory of the electromagnetic field in a variable-length one-dimensional cavity," *J. Math. Phys.*, vol. 11, pp. 2679–2691, 1970.

[119] C. Braggio, G. Bressi, G. Carugno, et al., "A novel experimental approach for the detection of the dynamical Casimir effect," *Europhys. Lett.*, vol. 70, pp. 754–760, 2005.

[120] L. H. Ford and A. Vilenkin, "Quantum radiation by moving mirrors," *Phys. Rev. D*, vol. 25, pp. 2569–2575, 1982.

[121] D. T. Alves, E. R. Granhen, and M. G. Lima, "Quantum radiation force on a moving mirror with Dirichlet and Neumann boundary conditions for a vacuum, finite temperature, and a coherent state," *Phys. Rev. D*, vol. 77, p. 125001, 2008.

[122] A. Lambrecht, M.-T. Jaekel, and S. Reynaud, "Motion induced radiation from a vibrating cavity," *Phys. Rev. Lett.*, vol. 77, pp. 615–618, 1996.

[123] P. A. M. Neto and L. A. S. Machado, "Quantum radiation generated by a moving mirror in free space," *Phys. Rev. A*, vol. 54, pp. 3420–3427, 1996.

[124] J. R. Johansson, G. Johansson, C. M. Wilson, and F. Nori, "Dynamical Casimir effect in superconducting microwave circuits," *Phys. Rev. A*, vol. 82, p. 052509, 2010.

[125] D. F. Mundarain and P. A. Maia Neto, "Quantum radiation in a plane cavity with moving mirrors," *Phys. Rev. A*, vol. 57, pp. 1379–1390, 1998.

[126] V. V. Dodonov, A. B. Klimov, and D. E. Nikonov, "Quantum phenomena in nonstationary media," *Phys. Rev. A*, vol. 47, pp. 4422–4429, 1993.

[127] M. Crocce, D. A. R. Dalvit, F. C. Lombardo, and F. D. Mazzitelli, "Model for resonant photon creation in a cavity with time-dependent conductivity," *Phys. Rev. A*, vol. 70, p. 033811, 2004.

[128] T. Kawakubo and K. Yamamoto, "Photon creation in a resonant cavity with a nonstationary plasma mirror and its detection with Rydberg atoms," *Phys. Rev. A*, vol. 83, p. 013819, 2011.

[129] M. Crocce, D. A. R. Dalvit, and F. D. Mazzitelli, "Resonant photon creation in a three-dimensional oscillating cavity," *Phys. Rev. A*, vol. 64, p. 013808, 2001.

[130] H. Johnston and S. Sarkar, "Moving mirrors and time-varying dielectrics," *Phys. Rev. A*, vol. 51, pp. 4109–4115, 1995.

[131] M. Uhlmann, G. Plunien, R. Schutzhold, and G. Soff, "Resonant cavity photon creation via the dynamical Casimir effect," *Phys. Rev. Lett.*, vol. 93, p. 193601, 2004.

[132] F. X. Dezael and A. Lambrecht, "Analogue Casimir radiation using an optical parametric oscillator," *Europhys. Lett.*, vol. 89, 2010, https://doi.org/10.1209/0295-5075/89/14001.

[133] D. Faccio and I. Carusotto, "Dynamical Casimir effect in optically modulated cavities," *Europhys. Lett.*, vol. 96, 2011, https://doi.org/10.1209/0295-5075/96/24006.

[134] P. Lähteenmäki, G. S. Paraoanu, J. Hassel, and P. J. Hakonen, "Dynamical Casimir effect in a Josephson metamaterial," *Proc. Natl. Acad. Sci. U. S. A.*, vol. 110, pp. 4234–4238, 2013.

[135] S. Vezzoli, A. Mussot, N. Westerberg, et al., "Optical analogue of the dynamical Casimir effect in a dispersion-oscillating fibre," *Commun. Phys.*, vol. 2, p. 84, 2019.

[136] J. A. Hutchison, T. Schwartz, C. Genet, E. Devaux, and T. W. Ebbesen, "Modifying chemical landscapes by coupling to vacuum fields," *Angew Chem. Int. Edit.*, vol. 51, pp. 1592–1596, 2012.

[137] A. Thomas, L. Lethuillier-Karl, K. Nagarajan, et al., "Tilting a ground-state reactivity landscape by vibrational strong coupling," *Science*, vol. 363, p. 616, 2019.

[138] J. P. Long and B. S. Simpkins, "Coherent coupling between a molecular vibration and Fabry-Perot optical cavity to give hybridized states in the strong coupling limit," *ACS Photonics*, vol. 2, pp. 130–136, 2015.

[139] A. Shalabney, J. George, J. Hutchison, G. Pupillo, C. Genet, and T. W. Ebbesen, "Coherent coupling of molecular resonators with a microcavity mode," *Nat. Commun.*, vol. 6, p. 5981, 2015.

[140] J. George, T. Chervy, A. Shalabney, et al., "Multiple Rabi splittings under ultrastrong vibrational coupling," *Phys. Rev. Lett.*, vol. 117, p. 153601, 2016.

[141] A. Thomas, J. George, A. Shalabney, et al., "Ground-state chemical reactivity under vibrational coupling to the vacuum electromagnetic field," *Angew Chem. Int. Edit.*, vol. 55, pp. 11462–11466, 2016.

[142] R. M. A. Vergauwe, J. George, T. Chervy, et al., "Quantum strong coupling with protein vibrational modes," *J. Phys. Chem. Lett.*, vol. 7, pp. 4159–4164, 2016.

[143] J. Bárcenas, L. Reyes, and R. Esquivel-Sirvent, "Scaling of micro- and nanodevices actuated by Casimir forces," *Appl. Phys. Lett.*, vol. 87, p. 263106, 2005.

[144] W. Broer, H. Waalkens, V. B. Svetovoy, J. Knoester, and G. Palasantzas, "Nonlinear actuation dynamics of driven Casimir oscillators with rough surfaces," *Phys. Rev. Appl.*, vol. 4, p. 054016, 2015.

[145] F. M. Serry, D. Walliser, and G. J. Maclay, "The anharmonic Casimir oscillator (ACO)-the Casimir effect in a model microelectromechanical system," *J. Microelectromech. Syst.*, vol. 4, pp. 193–205, 1995.

[146] E. Buks and M. L. Roukes, "Metastability and the Casimir effect in micromechanical systems," *Europhys. Lett.*, vol. 54, pp. 220–226, 2001.

[147] H. B. Chan, V. A. Aksyuk, R. N. Kleiman, D. J. Bishop, and F. Capasso, "Nonlinear micromechanical Casimir oscillator," *Phys. Rev. Lett.*, vol. 87, p. 211801, 2001.

[148] F. Tajik, M. Sedighi, A. A. Masoudi, H. Waalkens, and G. Palasantzas, "Sensitivity of chaotic behavior to low optical frequencies of a double-beam torsional actuator," *Phys. Rev. E*, vol. 100, p. 012201, 2019.

[149] D. Iannuzzi, J. N. Munday, and F. Capasso, "*Ultra-low friction configuration*," US Patent Application 20070066494, 2005.

[150] A. Stange, M. Imboden, J. Javor, L. K. Barrett, and D. J. Bishop, "Building a Casimir metrology platform with a commercial MEMS sensor," *Microsyst. Nanoeng.*, vol. 5, p. 14, 2019.

[151] M. Imboden, J. Morrison, D. K. Campbell, and D. J. Bishop, “Design of a Casimir-driven parametric amplifier,” *J. Appl. Phys.*, vol. 116, p. 134504, 2014.
[152] J. M. Pate, M. Goryachev, R. Y. Chiao, J. E. Sharping, and M. E. Tobar, “Casimir spring and dilution in macroscopic cavity optomechanics,” *Nat. Phys.*, 2020, https://doi.org/10.1038/s41567-020-0975-9.
[153] K. Y. Fong, H.-k. Li, R. Zhao, S. Yang, Y. Wang, and X. Zhang, “Phonon heat transfer across a vacuum through quantum fluctuations,” *Nature*, vol. 576, pp. 243–247, 2019.